p8. 3. Exercises Solutions

1. The product of 2 odd nos. is always a multiple of 3
e.g. 5 and 7, $5 \times 7 = 35$ which is not a multiple of 3

2. $\cos(-\theta) = -\cos\theta$
e.g. $\theta = 30°$, $\cos(-30) = \frac{\sqrt{3}}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$ but $-\cos 30 = -\frac{\sqrt{3}}{2} \neq \cos(-30)$
(In fact, $\cos(-\theta) = \cos\theta \ \forall \theta$)

3. $\sin(2\theta) = 2\sin\theta$
e.g. $\theta = 30°$, $\sin 2\theta = \sin 60 = \frac{\sqrt{3}}{2}$

$2\sin 30 = 2 \times 0.5 = 1 \neq \frac{\sqrt{3}}{2} = \sin 2\theta$

4. $\forall x \in \mathbb{R}, x^2 \geq x$
e.g. $x = \frac{1}{2}$, then $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} < \frac{1}{2}$

5. The product of 2 different irrational numbers is always irrational
e.g. $\sqrt{2}$ and $\sqrt{8}$, $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$ which is not irrational

6. $e^{2x} = e^x + e^2$
e.g. $x = 1$, $e^{2\times 1} = e^2 = 7.38\ldots$
$e^x + e^2 = e^1 + e^2 = 2.71\ldots + 7.38\ldots = 10.107\ldots \neq e^2$

7. For all positive values of a and b, $\ln(ab) = \ln a \ln b$
e.g. $a = 2, b = 3$ then $\ln(ab) = \ln(2\times 3) = \ln 6 = 1.79\ldots$
$\ln a \ln b = \ln 2 \times \ln 3 = 0.693\ldots \times 1.09\ldots = 0.761\ldots \neq 1.79\ldots$

8. $\forall x, y \in \mathbb{R}$, $x^2 > y^2 \Rightarrow x > y$
e.g. $x = -2$, $y = -1$ $x^2 = (-2)^2 = 4 > 1 = (-1)^2 = y^2$ but $-2 < -1$