

In[1]:= Needs ["SpinDynamica`"]

SpinDynamica version 3.0.1 loaded

**ModifyBuiltin:** The following built-in routines have been modified in SpinDynamica:  
{Chop, Dot, Duration, Exp, Expand, ExpandAll, NumericQ, Plus, Power, Simplify, Times, WignerD}.  
Evaluate ??symbol to generate the additional definitions for symbol.

## opR (rotation operators)

In[2]:= ? opR

opR[spins,angle] is the operator for rotating a single spin or a set of spins through the specified angle. If spins is absent, all spins in the current SpinSystem are rotated. The following formats for angle may be used: 1.  $\{\beta, \phi\}$  indicates a rotation through  $\beta$  about an axis in the xy plane with the phase  $\phi$ . The phase value may either be numeric, or be specified using the text codes "x", "y", "-x" for the quadrature phases. A code "z" indicates a rotation about the z-axis. 2. The format  $\{\xi, \{\theta, \phi\}\}$  indicates a rotation through the angle  $\xi$  about an axis with polar angles  $\theta$  and  $\phi$ . 3. The format  $\{\alpha, \beta, \gamma\}$  indicates a rotation through the specified Euler angles, using the zyz convention.

## 1 spins-1/2

In[3]:= SetSpinSystem[1]

**SetSpinSystem:** the spin system has been set to  $\left\{\left\{1, \frac{1}{2}\right\}\right\}$

**SetBasis:** the state basis has been set to ZeemanBasis $\left[\left\{\left\{1, \frac{1}{2}\right\}\right\}, \text{BasisLabels} \rightarrow \text{Automatic}\right]$ .

In[4]:= opR[{β, "x"}]

Out[4]=  $R_{1x}(\beta)$

In[5]:= MatrixRepresentation[opR[{β, "x"}]] // MatrixForm

Out[5]/MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\beta}{2}\right] & -i \sin\left[\frac{\beta}{2}\right] \\ -i \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right] \end{pmatrix}$$

In[6]:= MatrixRepresentation[opR[{β, φ}]] // MatrixForm

Out[6]/MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\beta}{2}\right] & -i e^{-i\phi} \sin\left[\frac{\beta}{2}\right] \\ -i e^{i\phi} \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right] \end{pmatrix}$$

In[7]:= MatrixRepresentation[opR[{α, β, γ}]] // MatrixForm

Out[7]/MatrixForm=

$$\begin{pmatrix} e^{-\frac{i\alpha}{2} - \frac{i\gamma}{2}} \cos\left[\frac{\beta}{2}\right] & -e^{-\frac{i\alpha}{2} + \frac{i\gamma}{2}} \sin\left[\frac{\beta}{2}\right] \\ e^{\frac{i\alpha}{2} - \frac{i\gamma}{2}} \sin\left[\frac{\beta}{2}\right] & e^{\frac{i\alpha}{2} + \frac{i\gamma}{2}} \cos\left[\frac{\beta}{2}\right] \end{pmatrix}$$

In[8]:= MatrixRepresentation[opR[{ξ, {θ, φ} }]] // MatrixForm

Out[8]/MatrixForm=

$$\begin{pmatrix} e^{\frac{i\phi}{2}} \left( e^{-\frac{i\xi}{2} - \frac{i\phi}{2}} \cos\left[\frac{\theta}{2}\right]^2 + e^{\frac{i\xi}{2} - \frac{i\phi}{2}} \sin\left[\frac{\theta}{2}\right]^2 \right) & e^{-\frac{i\phi}{2}} \left( e^{-\frac{i\xi}{2} - \frac{i\phi}{2}} \cos\left[\frac{\theta}{2}\right] \sin\left[\frac{\theta}{2}\right] - e^{\frac{i\xi}{2} - \frac{i\phi}{2}} \cos\left[\frac{\xi}{2}\right] \right) \\ e^{\frac{i\phi}{2}} \left( e^{-\frac{i\xi}{2} + \frac{i\phi}{2}} \cos\left[\frac{\theta}{2}\right] \sin\left[\frac{\theta}{2}\right] - e^{\frac{i\xi}{2} + \frac{i\phi}{2}} \cos\left[\frac{\theta}{2}\right] \sin\left[\frac{\theta}{2}\right] \right) & e^{-\frac{i\phi}{2}} \left( e^{\frac{i\xi}{2} + \frac{i\phi}{2}} \cos\left[\frac{\theta}{2}\right]^2 + e^{-\frac{i\xi}{2} + \frac{i\phi}{2}} \sin\left[\frac{\xi}{2}\right] \right) \end{pmatrix}$$

## 2 spins-1/2

In[9]:= **SetSpinSystem[2]**

... **SetSpinSystem**: the spin system has been set to  $\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$

... **SetBasis**: the state basis has been set to  $\text{ZeemanBasis}[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}, \text{BasisLabels} \rightarrow \text{Automatic}]$ .

In[10]:= **MatrixRepresentation[opR[{β, "x"}]] // MatrixForm**

Out[10]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\beta}{2}\right]^2 & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & -\sin\left[\frac{\beta}{2}\right]^2 \\ -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right]^2 & -\sin\left[\frac{\beta}{2}\right]^2 & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] \\ -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & -\sin\left[\frac{\beta}{2}\right]^2 & \cos\left[\frac{\beta}{2}\right]^2 & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] \\ -\sin\left[\frac{\beta}{2}\right]^2 & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & -i \cos\left[\frac{\beta}{2}\right] \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right]^2 \end{pmatrix}$$

In[11]:= **MatrixRepresentation[opR[1, {β, "x"}]] // MatrixForm**

Out[11]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\beta}{2}\right] & -i \sin\left[\frac{\beta}{2}\right] & 0 & 0 \\ -i \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right] & 0 & 0 \\ 0 & 0 & \cos\left[\frac{\beta}{2}\right] & -i \sin\left[\frac{\beta}{2}\right] \\ 0 & 0 & -i \sin\left[\frac{\beta}{2}\right] & \cos\left[\frac{\beta}{2}\right] \end{pmatrix}$$

In[12]:= **MatrixRepresentation[opR[2, {β, "x"}]] // MatrixForm**

Out[12]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\beta}{2}\right] & 0 & -i \sin\left[\frac{\beta}{2}\right] & 0 \\ 0 & \cos\left[\frac{\beta}{2}\right] & 0 & -i \sin\left[\frac{\beta}{2}\right] \\ -i \sin\left[\frac{\beta}{2}\right] & 0 & \cos\left[\frac{\beta}{2}\right] & 0 \\ 0 & -i \sin\left[\frac{\beta}{2}\right] & 0 & \cos\left[\frac{\beta}{2}\right] \end{pmatrix}$$

## 3 spins-1/2

In[13]:= **SetSpinSystem[3]**

... **SetSpinSystem**: the spin system has been set to  $\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}, \{3, \frac{1}{2}\}\}$

... **SetBasis**: the state basis has been set to  $\text{ZeemanBasis}[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}, \{3, \frac{1}{2}\}\}, \text{BasisLabels} \rightarrow \text{Automatic}]$ .

In[14]:= **Ispins = {1, 2}; Sspins = {3};**

In[15]:= **MatrixRepresentation**[opR[{ $\pi/2$ , "x"}]] // **MatrixForm**

Out[15]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} \\ -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} \\ -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} \\ -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} \\ \frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & -\frac{i}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix}$$

In[16]:= **MatrixRepresentation**[opR[Ispins, { $\pi/2$ , "x"}]] // **MatrixForm**

Out[16]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{i}{2} & -\frac{i}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 \\ -\frac{i}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{i}{2} & -\frac{i}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & -\frac{i}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{i}{2} \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{i}{2} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{i}{2} & -\frac{i}{2} & \frac{1}{2} \end{pmatrix}$$

In[17]:= **MatrixRepresentation**[opR[Sspins, { $\pi/2$ , "x"}]] // **MatrixForm**

Out[17]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$