In[1]:= `Needs["SpinDynamica`"]`

```
SpinDynamica version 3.0.1 loaded
```

ModifyBuiltIn: The following built-in routines have been modified in SpinDynamica:
{Chop, Dot, Duration, Exp, Expand, ExpandAll, NumericQ, Plus, Power, Simplify, Times, WignerD}.
Evaluate ??*symbol* to generate the additional definitions for *symbol*.

opI (angular momentum operators)

In[2]:= `? opI`

opI[spin, μ] defines an angular momentum operator for the indicated spin. If the symbol spin is a List, a sum over the spins in the list is implied, and if the symbol spin is absent, a sum over all spins in the current spin system is taken. The label μ may be a phase ϕ (indicating the phase angle of the spin angular momentum in the xy-plane), or may specify the polar angles $\{\theta,\phi\}$ (indicating spin angular momentum in an arbitrary direction in 3D space). The label μ may also take the values "x", "y", "z", indicating spin angular momentum operators along one of the Cartesian axes. Shift operators are indicated by using labels "+","-", while polarization operators for spins-1/2 are indicated by using labels "α", "β". If the label μ is absent, a vector of the three Cartesian angular momentum components is generated. Hence opI[i].opI[j] defines the dot product of the angular momenta of spins i and j.

2 spins-1/2

In[3]:= `SetSpinSystem[2]`

SetSpinSystem: the spin system has been set to $\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$

SetBasis: the state basis has been set to ZeemanBasis$[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$, BasisLabels → Automatic$]$.

In[4]:= `opI["x"]`

Out[4]= $I_{1x} + I_{2x}$

In[5]:= `opI[3, "x"]`

SpinSystem: spin index 3 is not in current spin system. Use SetSpinSystem or SetBasis to set up spin system.

Out[5]= `$Failed`

In[6]:= `opI[{1, 2}, "x"]`

Out[6]= $I_{1x} + I_{2x}$

In[7]:= `opI[{1, 2, 3}, "x"]`

SpinSystem: spin index {1, 2, 3} is not in current spin system. Use SetSpinSystem or SetBasis to set up spin system.

Out[7]= `$Failed`

In[8]:= `MatrixRepresentation[opI["x"]] // MatrixForm`

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

In[9]:=
```
MatrixRepresentation[opI[1, "x"]] // MatrixForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

In[10]:=
```
MatrixRepresentation[opI[2, "x"]] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

In[11]:=
```
MatrixRepresentation[2 opI[1, "z"].opI[2, "x"]] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

In[12]:=
```
MatrixRepresentation[2 opI[1, "z"].opI[2, "+"]] // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[13]:=
```
MatrixRepresentation[opI[1, "α"].opI[2, "β"]] // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[14]:=
```
opI[1, π / 4]
```

Out[14]= $\frac{I_{1x}}{\sqrt{2}} + \frac{I_{1y}}{\sqrt{2}}$

In[15]:=
```
opI[1, {π/3, π / 4}]
```

Out[15]= $\frac{1}{2}\sqrt{3}\left(\frac{I_{1x}}{\sqrt{2}} + \frac{I_{1y}}{\sqrt{2}}\right) + \frac{I_{1z}}{2}$

In[16]:=
```
opI[{π/3, π / 4}]
```

Out[16]= $\frac{1}{2}\sqrt{3}\left(\frac{I_{1x}}{\sqrt{2}} + \frac{I_{1y}}{\sqrt{2}}\right) + \frac{I_{1z}}{2} + \frac{1}{2}\sqrt{3}\left(\frac{I_{2x}}{\sqrt{2}} + \frac{I_{2y}}{\sqrt{2}}\right) + \frac{I_{2z}}{2}$

In[17]:=
```
opI[1].opI[2]
```

Out[17]= $I_{1x} \bullet I_{2x} + I_{1y} \bullet I_{2y} + I_{1z} \bullet I_{2z}$

In[18]:=
```
opI[1].{a, b, c}
```

Out[18]= $a\, I_{1\,x} + b\, I_{1\,y} + c\, I_{1\,z}$

spin "I" (quantum number 1) and spin "S" (quantum number 3/2)

In[19]:=
```
SetSpinSystem[{{"I", 1}, {"S", 3/2}}]
```

SetSpinSystem: the spin system has been set to $\{\{I, 1\}, \{S, \frac{3}{2}\}\}$

SetBasis: the state basis has been set to ZeemanBasis[$\{\{I, 1\}, \{S, \frac{3}{2}\}\}$, BasisLabels → Automatic].

In[20]:=
```
opI["x"]
```

Out[20]= $I_x + S_x$

In[21]:=
```
MatrixRepresentation[opI["x"]] // MatrixForm
```

Out[21]//MatrixForm=

$$\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{3}}{2} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{3}}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{\sqrt{3}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{\sqrt{3}}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & \frac{1}{\sqrt{2}} & 0
\end{pmatrix}$$

In[22]:=
```
opI[1, "x"]
```

SpinSystem: spin index 1 is not in current spin system. Use SetSpinSystem or SetBasis to set up spin system.

Out[22]= $Failed

In[23]:= `MatrixRepresentation[opI["I", "x"]] // MatrixForm`

Out[23]//MatrixForm=

$$\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0
\end{pmatrix}$$

In[24]:= `MatrixRepresentation[opI["S", "x"]] // MatrixForm`

Out[24]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0
\end{pmatrix}$$

In[25]:=
```
MatrixRepresentation[opI["I", "z"]^2] // MatrixForm
```

Out[25]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1

In[26]:=
```
? AngularMomentumOperator
```

AngularMomentumOperator is a synonym for opI

In[27]:=
```
MatrixRepresentation[AngularMomentumOperator["I", "z"]^2] // MatrixForm
```

Out[27]//MatrixForm=

1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1