In[57]:= `Needs["SpinDynamica`"]`

# Wignerd

In[58]:= `?Wignerd`

Wignerd[J][β] is the reduced Wigner matrix (J integral or half–integral). Wignerd[J,{m2,m1}][β] is the reduced Wigner function (element of the reduced Wigner matrix). Wignerd[J,{m}][β] is the mth row of the reduced Wigner matrix; Wignerd[J,{{m}}][β] is the mth column of the reduced Wigner matrix. In all cases the row and column indices run from +J to –J in descending order.

# rank 0

In[59]:= `Wignerd[0][β] // MatrixForm`

Out[59]//MatrixForm=

$$\begin{pmatrix} 1 \end{pmatrix}$$

In[60]:= `Wignerd[0, {0, 0}][β]`

Out[60]= 1

# rank 1

In[61]:= `Wignerd[1][β] // MatrixForm`

Out[61]//MatrixForm=

$$\begin{pmatrix} \text{Cos}\left[\frac{\beta}{2}\right]^2 & -\sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right] & \text{Sin}\left[\frac{\beta}{2}\right]^2 \\ \sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right] & \text{Cos}[\beta] & -\sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right] \\ \text{Sin}\left[\frac{\beta}{2}\right]^2 & \sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right] & \text{Cos}\left[\frac{\beta}{2}\right]^2 \end{pmatrix}$$

In[62]:= `Wignerd[1, {0, 0}][β]`

Out[62]= $\text{Cos}[\beta]$

In[63]:= `Wignerd[1, {0}][β]`

Out[63]= $\left\{\sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right], \text{Cos}[\beta], -\sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right]\right\}$

In[64]:= `Wignerd[1, {{0}}][β]`

Out[64]= $\left\{-\sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right], \text{Cos}[\beta], \sqrt{2}\,\text{Cos}\left[\frac{\beta}{2}\right]\text{Sin}\left[\frac{\beta}{2}\right]\right\}$

# rank 2

In[65]:= `Wignerd[2][β] // MatrixForm`

Out[65]//MatrixForm=

$$\left(\begin{array}{cc}
\text{Cos}\left[\frac{\beta}{2}\right]^4 & -\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]-\text{Cos}\cdots \\
\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]+\text{Cos}\left[\frac{\beta}{2}\right]^3\,\text{Sin}\left[\frac{\beta}{2}\right]-\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]^3 & \frac{1}{2}\,\text{Cos}\left[\frac{\beta}{2}\right]^2+\frac{1}{2}\,\text{Cos}\left[\frac{\beta}{2}\right]^4-\frac{1}{2}\,\text{Sin}\cdots \\
\frac{\sqrt{\frac{3}{2}}}{4}-\frac{1}{4}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^2\text{Sin}\left[\frac{\beta}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\,\text{Sin}\left[\frac{\beta}{2}\right]^4 & \sqrt{6}\,\text{Cos}\left[\frac{\beta}{2}\right]^3\,\text{Sin}\left[\cdots\right. \\
\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]-\text{Cos}\left[\frac{\beta}{2}\right]^3\,\text{Sin}\left[\frac{\beta}{2}\right]+\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]^3 & \frac{1}{2}\,\text{Cos}\left[\frac{\beta}{2}\right]^2-\frac{1}{2}\,\text{Cos}\left[\frac{\beta}{2}\right]^4-\frac{1}{2}\,\text{Sin}\cdots \\
\text{Sin}\left[\frac{\beta}{2}\right]^4 & \text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]-\text{Cos}\left[\cdots\right.
\end{array}\right.$$

In[66]:= `Wignerd[2, {0, 0}][β]`

Out[66]= $-\frac{1}{2}+\frac{3\,\text{Cos}[\beta]^2}{2}$

# rank 3

In[67]:= `Wignerd[3][β] // MatrixForm`

Out[67]//MatrixForm=

$$\left(\begin{array}{c}
\text{Cos}\cdots \\
\frac{5}{8}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]+\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^3\,\text{Sin}\left[\frac{\beta}{2}\right]+\frac{3}{8}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^5\,\text{Sin}\left[\frac{\beta}{2}\right]\cdots \\
\frac{\sqrt{15}}{16}+\frac{1}{32}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^2-\frac{1}{16}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^4-\frac{1}{32}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^6-\frac{1}{32}\sqrt{15}\,\text{Sin}\left[\frac{\beta}{2}\right]^2+\frac{3}{8}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^2\cdots \\
\frac{3}{8}\sqrt{5}\,\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]-\frac{3}{8}\sqrt{5}\,\text{Cos}\left[\frac{\beta}{2}\right]^5\,\text{Sin}\left[\frac{\beta}{2}\right]\cdots \\
\frac{\sqrt{15}}{16}-\frac{1}{32}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^2-\frac{1}{16}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^4+\frac{1}{32}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^6+\frac{1}{32}\sqrt{15}\,\text{Sin}\left[\frac{\beta}{2}\right]^2+\frac{3}{8}\sqrt{15}\,\text{Cos}\left[\frac{\beta}{2}\right]^2\cdots \\
\frac{5}{8}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]\,\text{Sin}\left[\frac{\beta}{2}\right]-\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^3\,\text{Sin}\left[\frac{\beta}{2}\right]+\frac{3}{8}\sqrt{\frac{3}{2}}\,\text{Cos}\left[\frac{\beta}{2}\right]^5\,\text{Sin}\left[\frac{\beta}{2}\right]\cdots \\
\text{Sin}\cdots
\end{array}\right.$$

In[68]:= `Wignerd[3, {0, 0}][β]`

Out[68]= $\frac{3\,\text{Cos}[\beta]}{8}+\frac{5\,\text{Cos}[\beta]^3}{8}-\frac{15}{8}\,\text{Cos}[\beta]\,\text{Sin}[\beta]^2$

# WignerD

In[69]:= `?WignerD`

WignerD is a built–in function in Mathematica 2.0 but has been given enhanced functionality in SpinDynamica, in order to maintain compatibility with previous code. The SpinDynamica definitions are as follows: WignerD[J][{α,β,γ}] is the three–angle Wigner matrix for integral or half–integral J. WignerD[J][{α2,β2,γ2},{α1,β1,γ1}..}] is the Wigner matrix for the overall rotation corresponding to the specified sequence of Euler rotations. WignerD[J,{m2,m1}][{α,β,γ}] is a three–angle Wigner function (Wigner matrix element). WignerD[J,{m}][{α,β,γ}] is the mth row of the Wigner matrix; WignerD[J,{{m}}][{α,β,γ}] is the mth column of the Wigner matrix. In all cases the row and column indices run from +J to –J in descending order. >>

## single Euler rotation

In[70]:= `euler = {α, β, γ}`

Out[70]= {α, β, γ}

---

# rank 0

In[71]:= `WignerD[0][euler] // MatrixForm`

Out[71]//MatrixForm=

$$\begin{pmatrix} 1 \end{pmatrix}$$

In[72]:= `WignerD[0, {0, 0}][euler]`

Out[72]= 1

---

# rank 1

In[73]:= `WignerD[1][euler] // MatrixForm`

Out[73]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} e^{-i\alpha-i\gamma} + \frac{1}{2} e^{-i\alpha-i\gamma} \text{Cos}\left[\frac{\beta}{2}\right]^2 - \frac{1}{2} e^{-i\alpha-i\gamma} \text{Sin}\left[\frac{\beta}{2}\right]^2 & -\sqrt{2}\, e^{-i\alpha} \text{Cos}\left[\frac{\beta}{2}\right] \text{Sin}\left[\frac{\beta}{2}\right] & \frac{1}{2} e^{-i\alpha+i\gamma} - \frac{1}{2} e^{-i\alpha+i} \\ \sqrt{2}\, e^{-i\gamma} \text{Cos}\left[\frac{\beta}{2}\right] \text{Sin}\left[\frac{\beta}{2}\right] & \text{Cos}[\beta] & -\sqrt{2}\, e \\ \frac{1}{2} e^{i\alpha-i\gamma} - \frac{1}{2} e^{i\alpha-i\gamma} \text{Cos}\left[\frac{\beta}{2}\right]^2 + \frac{1}{2} e^{i\alpha-i\gamma} \text{Sin}\left[\frac{\beta}{2}\right]^2 & \sqrt{2}\, e^{i\alpha} \text{Cos}\left[\frac{\beta}{2}\right] \text{Sin}\left[\frac{\beta}{2}\right] & \frac{1}{2} e^{i\alpha+i\gamma} + \frac{1}{2} e^{i\alpha+i} \end{pmatrix}$$

In[74]:= `WignerD[1, {0, 0}][euler]`

Out[74]= Cos[β]

# rank 2

In[75]:= `WignerD[2][euler] // MatrixForm`

Out[75]//MatrixForm=

$$\frac{3}{8} e^{-2 i \alpha-2 i \gamma}+\frac{1}{2} e^{-2 i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^2+\frac{1}{8} e^{-2 i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^4-\frac{1}{2} e^{-2 i \alpha-2 i \gamma} \mathrm{Sin}\left[\frac{\beta}{2}\right]^2-\frac{3}{4} e^{-2 i \alpha-2 i \gamma} \mathrm{Cos}[$$

$$e^{-i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right] \mathrm{Sin}\left[\frac{\beta}{2}\right]+e^{-i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^3 \mathrm{Sin}\left[\frac{\beta}{2}\right]-e^{-i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]$$

$$\frac{1}{4} \sqrt{\frac{3}{2}} e^{-2 i \gamma}-\frac{1}{4} \sqrt{\frac{3}{2}} e^{-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^4+\frac{3}{2} \sqrt{\frac{3}{2}} e^{-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^2 \mathrm{Sin}\left[\frac{\beta}{2}\right]^2-\frac{1}{4} \sqrt{\frac{3}{2}}$$

$$e^{i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right] \mathrm{Sin}\left[\frac{\beta}{2}\right]-e^{i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^3 \mathrm{Sin}\left[\frac{\beta}{2}\right]+e^{i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right] \mathrm{S}$$

$$\frac{3}{8} e^{2 i \alpha-2 i \gamma}-\frac{1}{2} e^{2 i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^2+\frac{1}{8} e^{2 i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^4+\frac{1}{2} e^{2 i \alpha-2 i \gamma} \mathrm{Sin}\left[\frac{\beta}{2}\right]^2-\frac{3}{4} e^{2 i \alpha-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right.$$

In[76]:= `WignerD[2, {0, 0}][euler]`

Out[76]= $$-\frac{1}{2}+\frac{3 \mathrm{Cos}[\beta]^2}{2}$$

In[77]:= `WignerD[2, {0}][euler]`

Out[77]= $$\left\{\frac{1}{4} \sqrt{\frac{3}{2}} e^{-2 i \gamma}-\frac{1}{4} \sqrt{\frac{3}{2}} e^{-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^4+\frac{3}{2} \sqrt{\frac{3}{2}} e^{-2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^2 \mathrm{Sin}\left[\frac{\beta}{2}\right]^2-\right.$$

$$\frac{1}{4} \sqrt{\frac{3}{2}} e^{-2 i \gamma} \mathrm{Sin}\left[\frac{\beta}{2}\right]^4, \sqrt{6} e^{-i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^3 \mathrm{Sin}\left[\frac{\beta}{2}\right]-\sqrt{6} e^{-i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right] \mathrm{Sin}\left[\frac{\beta}{2}\right]^3,$$

$$-\frac{1}{2}+\frac{3 \mathrm{Cos}[\beta]^2}{2}, -\sqrt{6} e^{i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^3 \mathrm{Sin}\left[\frac{\beta}{2}\right]+\sqrt{6} e^{i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right] \mathrm{Sin}\left[\frac{\beta}{2}\right]^3,$$

$$\left.\frac{1}{4} \sqrt{\frac{3}{2}} e^{2 i \gamma}-\frac{1}{4} \sqrt{\frac{3}{2}} e^{2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^4+\frac{3}{2} \sqrt{\frac{3}{2}} e^{2 i \gamma} \mathrm{Cos}\left[\frac{\beta}{2}\right]^2 \mathrm{Sin}\left[\frac{\beta}{2}\right]^2-\frac{1}{4} \sqrt{\frac{3}{2}} e^{2 i \gamma} \mathrm{Sin}\left[\frac{\beta}{2}\right]^4\right\}$$

# rank 3

## sequences of Euler rotations

In[80]:= `euler3 = {α3, β3, γ3}; euler2 = {α2, β2, γ2}; euler1 = {α1, β1, γ1};`

In[81]:= `WignerD[2][{euler2, euler1}] // MatrixForm`

Out[81]//MatrixForm=

( ⋯ 1 ⋯ )

large output | show less | show more | show all | set size limit...

In[82]:= `WignerD[2, {0, 0}][{euler2, euler1}]`

Out[82]=
$$\left(-\frac{1}{2}+\frac{3\,\text{Cos}[\beta 1]^2}{2}\right)\left(-\frac{1}{2}+\frac{3\,\text{Cos}[\beta 2]^2}{2}\right)+$$
$$\left(-\sqrt{6}\,e^{-i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^3\text{Sin}\left[\frac{\beta 1}{2}\right]+\sqrt{6}\,e^{-i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]\text{Sin}\left[\frac{\beta 1}{2}\right]^3\right)$$
$$\left(\sqrt{6}\,e^{-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]-\sqrt{6}\,e^{-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]^3\right)+$$
$$\left(\sqrt{6}\,e^{i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^3\text{Sin}\left[\frac{\beta 1}{2}\right]-\sqrt{6}\,e^{i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]\text{Sin}\left[\frac{\beta 1}{2}\right]^3\right)$$
$$\left(-\sqrt{6}\,e^{i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]+\sqrt{6}\,e^{i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]^3\right)+$$
$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\alpha 1}-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\,e^{-2i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^2\text{Sin}\left[\frac{\beta 1}{2}\right]^2-\right.$$
$$\left.\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\alpha 1}\,\text{Sin}\left[\frac{\beta 1}{2}\right]^4\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\gamma 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4+\right.$$
$$\left.\frac{3}{2}\sqrt{\frac{3}{2}}\,e^{-2i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{-2i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\right)+$$
$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\alpha 1}-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\,e^{2i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^2\text{Sin}\left[\frac{\beta 1}{2}\right]^2-\right.$$
$$\left.\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\alpha 1}\,\text{Sin}\left[\frac{\beta 1}{2}\right]^4\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\gamma 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4+\right.$$
$$\left.\frac{3}{2}\sqrt{\frac{3}{2}}\,e^{2i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\,e^{2i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\right)$$

In[83]:= `WignerD[2, {0, 0}][{euler3, euler2, euler1}]`

Out[83]=
$$\left(-\sqrt{6}\,e^{-i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^3\text{Sin}\left[\frac{\beta 1}{2}\right]+\sqrt{6}\,e^{-i\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]\text{Sin}\left[\frac{\beta 1}{2}\right]^3\right)$$
$$\left(\left(-\frac{1}{2}+\frac{3\,\text{Cos}[\beta 3]^2}{2}\right)\left(\sqrt{6}\,e^{-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]-\sqrt{6}\,e^{-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]^3\right)+\right.$$
$$\left(\frac{1}{2}e^{-i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{2}e^{-i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4-\frac{1}{2}e^{-i\alpha 2-i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\right.$$
$$\left.3\,e^{-i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{2}e^{-i\alpha 2-i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\right)$$
$$\left(\sqrt{6}\,e^{-i\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^3\text{Sin}\left[\frac{\beta 3}{2}\right]-\sqrt{6}\,e^{-i\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]\text{Sin}\left[\frac{\beta 3}{2}\right]^3\right)+$$
$$\left(\frac{1}{2}e^{i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{2}e^{i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4-\frac{1}{2}e^{i\alpha 2-i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^2+\right.$$
$$\left.3\,e^{i\alpha 2-i\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{2}e^{i\alpha 2-i\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\right)$$

$$\left(-\sqrt{6}\ e^{i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 3}{2}\right]+\sqrt{6}\ e^{i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^3\right)+$$

$$\left(-e^{-2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]-e^{-2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 2}{2}\right]+\right.$$

$$\left.e^{-2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^3\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^4+\right.$$

$$\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^2\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^4\right)+$$

$$\left(e^{2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]-e^{2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 2}{2}\right]+e^{2i\alpha 2-i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^3\right)$$

$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2i\gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^4+\right.$$

$$\left.\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^2\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2i\gamma 3}\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^4\right)\right)+$$

$$\left(\sqrt{6}\ e^{i\alpha 1}\operatorname{Cos}\left[\frac{\beta 1}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 1}{2}\right]-\sqrt{6}\ e^{i\alpha 1}\operatorname{Cos}\left[\frac{\beta 1}{2}\right]\operatorname{Sin}\left[\frac{\beta 1}{2}\right]^3\right)$$

$$\left(\left(-\frac{1}{2}+\frac{3\operatorname{Cos}[\beta 3]^2}{2}\right)\left(-\sqrt{6}\ e^{i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 2}{2}\right]+\sqrt{6}\ e^{i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^3\right)+\right.$$

$$\left(\frac{1}{2}e^{-i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{2}e^{-i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^4-\frac{1}{2}e^{-i\alpha 2+i\gamma 2}\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^2+\right.$$

$$\left.3\,e^{-i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^2\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{2}e^{-i\alpha 2+i\gamma 2}\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^4\right)$$

$$\left(\sqrt{6}\ e^{-i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 3}{2}\right]-\sqrt{6}\ e^{-i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^3\right)+$$

$$\left(\frac{1}{2}e^{i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{2}e^{i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^4-\frac{1}{2}e^{i\alpha 2+i\gamma 2}\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^2-\right.$$

$$\left.3\,e^{i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^2\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{2}e^{i\alpha 2+i\gamma 2}\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^4\right)$$

$$\left(-\sqrt{6}\ e^{i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 3}{2}\right]+\sqrt{6}\ e^{i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^3\right)+$$

$$\left(-e^{-2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]+e^{-2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 2}{2}\right]-\right.$$

$$\left.e^{-2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^3\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^4+\right.$$

$$\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Cos}\left[\frac{\beta 3}{2}\right]^2\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2i\gamma 3}\operatorname{Sin}\left[\frac{\beta 3}{2}\right]^4\right)+$$

$$\left(e^{2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]+e^{2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]^3\operatorname{Sin}\left[\frac{\beta 2}{2}\right]-e^{2i\alpha 2+i\gamma 2}\operatorname{Cos}\left[\frac{\beta 2}{2}\right]\operatorname{Sin}\left[\frac{\beta 2}{2}\right]^3\right)$$

$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{4}+\right.$$

$$\left.\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{4}\right)\right)+$$

$$\left(-\frac{1}{2}+\frac{3\,\mathrm{Cos}[\beta 1]^{2}}{2}\right)\left(\left(-\frac{1}{2}+\frac{3\,\mathrm{Cos}[\beta 2]^{2}}{2}\right)\left(-\frac{1}{2}+\frac{3\,\mathrm{Cos}[\beta 3]^{2}}{2}\right)+\right.$$

$$\left(-\sqrt{6}\ e^{-i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{3}\mathrm{Sin}\left[\frac{\beta 2}{2}\right]+\sqrt{6}\ e^{-i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{3}\right)$$

$$\left(\sqrt{6}\ e^{-i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{3}\mathrm{Sin}\left[\frac{\beta 3}{2}\right]-\sqrt{6}\ e^{-i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{3}\right)+$$

$$\left(\sqrt{6}\ e^{i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{3}\mathrm{Sin}\left[\frac{\beta 2}{2}\right]-\sqrt{6}\ e^{i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{3}\right)$$

$$\left(-\sqrt{6}\ e^{i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{3}\mathrm{Sin}\left[\frac{\beta 3}{2}\right]+\sqrt{6}\ e^{i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{3}\right)+$$

$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{4}+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}-\right.$$

$$\left.\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{4}\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{4}+\right.$$

$$\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{4}\right)+$$

$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \alpha 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{4}+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2 i \alpha 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}-\right.$$

$$\left.\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \alpha 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{4}\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{4}+\right.$$

$$\left.\left.\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2 i \gamma 3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{4}\right)\right)+$$

$$\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 1}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 1}\,\mathrm{Cos}\left[\frac{\beta 1}{2}\right]^{4}+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 1}\,\mathrm{Cos}\left[\frac{\beta 1}{2}\right]^{2}\mathrm{Sin}\left[\frac{\beta 1}{2}\right]^{2}-\right.$$

$$\left.\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \alpha 1}\,\mathrm{Sin}\left[\frac{\beta 1}{2}\right]^{4}\right)$$

$$\left(\left(-\frac{1}{2}+\frac{3\,\mathrm{Cos}[\beta 3]^{2}}{2}\right)\left(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2 i \gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{4}+\right.\right.$$

$$\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\Bigg)+$$

$$\Bigg(e^{-i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]+e^{-i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]-e^{-i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]$$

$$\text{Sin}\left[\frac{\beta 2}{2}\right]^3\Bigg)\ \Bigg(\sqrt{6}\ e^{-i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^3\text{Sin}\left[\frac{\beta 3}{2}\right]-\sqrt{6}\ e^{-i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]\text{Sin}\left[\frac{\beta 3}{2}\right]^3\Bigg)+$$

$$\Bigg(e^{i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]-e^{i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]+e^{i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]^3\Bigg)$$

$$\Bigg(-\sqrt{6}\ e^{i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^3\text{Sin}\left[\frac{\beta 3}{2}\right]+\sqrt{6}\ e^{i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]\text{Sin}\left[\frac{\beta 3}{2}\right]^3\Bigg)+$$

$$\Bigg(\frac{3}{8}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}+\frac{1}{2}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{8}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4-$$

$$\frac{1}{2}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{3}{4}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{8}\,e^{-2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\Bigg)$$

$$\Bigg(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 3}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^2\text{Sin}\left[\frac{\beta 3}{2}\right]^2-$$

$$\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{-2\,i\,\gamma 3}\,\text{Sin}\left[\frac{\beta 3}{2}\right]^4\Bigg)+$$

$$\Bigg(\frac{3}{8}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}-\frac{1}{2}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{8}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4+\frac{1}{2}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^2-$$

$$\frac{3}{4}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2+\frac{1}{8}\,e^{2\,i\,\alpha 2-2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\Bigg)\ \Bigg(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 3}-$$

$$\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^2\text{Sin}\left[\frac{\beta 3}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 3}\,\text{Sin}\left[\frac{\beta 3}{2}\right]^4\Bigg)\Bigg)+$$

$$\Bigg(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\alpha 1}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^4+\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2\,i\,\alpha 1}\,\text{Cos}\left[\frac{\beta 1}{2}\right]^2\text{Sin}\left[\frac{\beta 1}{2}\right]^2-$$

$$\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\alpha 1}\,\text{Sin}\left[\frac{\beta 1}{2}\right]^4\Bigg)$$

$$\Bigg(\Bigg(-\frac{1}{2}+\frac{3\,\text{Cos}[\beta 3]^2}{2}\Bigg)\ \Bigg(\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 2}-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^4+$$

$$\frac{3}{2}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^2\text{Sin}\left[\frac{\beta 2}{2}\right]^2-\frac{1}{4}\sqrt{\frac{3}{2}}\ e^{2\,i\,\gamma 2}\,\text{Sin}\left[\frac{\beta 2}{2}\right]^4\Bigg)+$$

$$\Bigg(-e^{-i\,\alpha 2+2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]\text{Sin}\left[\frac{\beta 2}{2}\right]+e^{-i\,\alpha 2+2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]^3\text{Sin}\left[\frac{\beta 2}{2}\right]-e^{-i\,\alpha 2+2\,i\,\gamma 2}\,\text{Cos}\left[\frac{\beta 2}{2}\right]$$

$$\text{Sin}\left[\frac{\beta 2}{2}\right]^3\Bigg)\ \Bigg(\sqrt{6}\ e^{-i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]^3\text{Sin}\left[\frac{\beta 3}{2}\right]-\sqrt{6}\ e^{-i\,\gamma 3}\,\text{Cos}\left[\frac{\beta 3}{2}\right]\text{Sin}\left[\frac{\beta 3}{2}\right]^3\Bigg)+$$

$$\left(-e^{i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]-e^{i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{3}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]+e^{i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]\right.$$

$$\left.\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{3}\right)\,\left(-\sqrt{6}\,\,e^{i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]+\sqrt{6}\,\,e^{i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{3}\right)+$$

$$\left(\frac{3}{8}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}-\frac{1}{2}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}+\frac{1}{8}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{4}+\right.$$

$$\left.\frac{1}{2}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}-\frac{3}{4}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}+\frac{1}{8}\,e^{-2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{4}\right)$$

$$\left(\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{-2\,i\,\gamma 3}-\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{-2\,i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{4}+\frac{3}{2}\,\sqrt{\frac{3}{2}}\,\,e^{-2\,i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{2}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{2}-\right.$$

$$\left.\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{-2\,i\,\gamma 3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{4}\right)+$$

$$\left(\frac{3}{8}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}+\frac{1}{2}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}+\frac{1}{8}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{4}-\frac{1}{2}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}-\right.$$

$$\left.\frac{3}{4}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Cos}\left[\frac{\beta 2}{2}\right]^{2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{2}+\frac{1}{8}\,e^{2\,i\,\alpha 2+2\,i\,\gamma 2}\,\mathrm{Sin}\left[\frac{\beta 2}{2}\right]^{4}\right)\,\left(\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{2\,i\,\gamma 3}-\right.$$

$$\left.\left.\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{2\,i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{4}+\frac{3}{2}\,\sqrt{\frac{3}{2}}\,\,e^{2\,i\,\gamma 3}\,\mathrm{Cos}\left[\frac{\beta 3}{2}\right]^{2}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{2}-\frac{1}{4}\,\sqrt{\frac{3}{2}}\,\,e^{2\,i\,\gamma 3}\,\mathrm{Sin}\left[\frac{\beta 3}{2}\right]^{4}\right)\right)$$