In[1]:= `Needs["SpinDynamica`"]`

```
SpinDynamica version 3.0.1 loaded
```

ModifyBuiltIn: The following built-in routines have been modified in SpinDynamica:
{Chop, Dot, Duration, Exp, Expand, ExpandAll, NumericQ, Plus, Power, Simplify, Times, WignerD}.
Evaluate ??*symbol* to generate the additional definitions for *symbol*.

In[2]:= `SetSpinSystem[2]`

SetSpinSystem: the spin system has been set to $\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$

SetBasis: the state basis has been set to ZeemanBasis$[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$, BasisLabels → Automatic$]$.

default operator basis (ZeemanKetBraOperatorBasis)

natural sorting order

In[3]:= `SetOperatorBasis[]`

SetOperatorBasis: the operator basis has been set to ShiftAndZOperatorBasis$[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$, Sorted → CoherenceOrder$]$.

In[4]:= `BasisOperators[]`

Out[4]= $\{I_1^- \cdot I_2^-, \sqrt{2}\,(I_1^- \cdot I_{2z}), \sqrt{2}\,(I_{1z} \cdot I_2^-), \frac{I_1^-}{\sqrt{2}}, \frac{I_2^-}{\sqrt{2}}, I_1^- \cdot I_2^+, I_1^+ \cdot I_2^-, 2\,(I_{1z} \cdot I_{2z}), I_{1z}, I_{2z}, \frac{\mathbb{1}}{2}, \sqrt{2}\,(I_1^+ \cdot I_{2z}), \sqrt{2}\,(I_{1z} \cdot I_2^+), \frac{I_1^+}{\sqrt{2}}, \frac{I_2^+}{\sqrt{2}}, I_1^+ \cdot I_2^+\}$

In[5]:= `CoherenceOrder /@ BasisOperators[]`

Out[5]= {-2, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2}

sort the basis operators according to the coherence order

In[6]:= `SetOperatorBasis[ZeemanKetBraOperatorBasis[Sorted → CoherenceOrder]]`

SetOperatorBasis: the operator basis has been set to
ZeemanKetBraOperatorBasis$[\{\{1, \frac{1}{2}\}, \{2, \frac{1}{2}\}\}$, Sorted → CoherenceOrder$]$.

In[7]:= `BasisOperators[]`

Out[7]= {|ββ⟩.⟨αα|, |ββ⟩.⟨βα|, |ββ⟩.⟨αβ|, |βα⟩.⟨αα|, |αβ⟩.⟨αα|, |ββ⟩.⟨ββ|, |βα⟩.⟨βα|, |βα⟩.⟨αβ|, |αβ⟩.⟨βα|, |αβ⟩.⟨αβ|, |αα⟩.⟨αα|, |βα⟩.⟨ββ|, |αβ⟩.⟨ββ|, |αα⟩.⟨βα|, |αα⟩.⟨αβ|, |αα⟩.⟨ββ|}

In[8]:= `CoherenceOrder /@ BasisOperators[]`

Out[8]= {-2, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2}

In[9]:=
```
OperatorBasisTransformationMatrix[
  ZeemanKetBraOperatorBasis[Sorted → CoherenceOrder],
  ZeemanKetBraOperatorBasis[Sorted → False]
 ] // MatrixForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

CartesianProductOperatorBasis

default sorting

In[10]:=
```
SetOperatorBasis[CartesianProductOperatorBasis[]]
```

SetOperatorBasis: the operator basis has been set to

$\text{CartesianProductOperatorBasis}\left[\left\{\left\{1, \frac{1}{2}\right\}, \left\{2, \frac{1}{2}\right\}\right\}, \text{Sorted} \rightarrow \text{SpinProductRank}\right]$.

In[11]:=
```
BasisOperators[]
```

Out[11]= $\left\{\frac{\mathbb{1}}{2}, I_{1x}, I_{1y}, I_{1z}, I_{2x}, I_{2y}, I_{2z}, 2\,(I_{1x} \bullet I_{2x}), 2\,(I_{1x} \bullet I_{2y}), 2\,(I_{1x} \bullet I_{2z}), 2\,(I_{1y} \bullet I_{2x}), 2\,(I_{1y} \bullet I_{2y}), 2\,(I_{1y} \bullet I_{2z}), 2\,(I_{1z} \bullet I_{2x}), 2\,(I_{1z} \bullet I_{2y}), 2\,(I_{1z} \bullet I_{2z})\right\}$

In[12]:=
```
CoherenceOrder /@ BasisOperators[]
```

Out[12]= {0, {-1, 1}, {-1, 1}, 0, {-1, 1}, {-1, 1}, 0, {-2, 0, 2}, {-2, 0, 2}, {-1, 1}, {-2, 0, 2}, {-2, 0, 2}, {-1, 1}, {-1, 1}, {-1, 1}, 0}

In[13]:=
```
SpinProductRank /@ BasisOperators[]
```

Out[13]= {0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2}

basis transformation matrix

In[14]:=
```
OperatorBasisTransformationMatrix[
   ZeemanKetBraOperatorBasis[Sorted → CoherenceOrder],
   CartesianProductOperatorBasis[]
  ] // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & \frac{i}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{i}{2} & 0 \\
0 & \frac{1}{2} & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 \\
\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & \frac{i}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & -\frac{i}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{i}{2} & 0 \\
0 & \frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0
\end{pmatrix}$$

ShiftAndZOperatorBasis

default sorting

In[15]:=
```
SetOperatorBasis[ShiftAndZOperatorBasis[]]
```

SetOperatorBasis: the operator basis has been set to ShiftAndZOperatorBasis$\left[\left\{\left\{1, \frac{1}{2}\right\}, \left\{2, \frac{1}{2}\right\}\right\}, \text{Sorted} \to \text{CoherenceOrder}\right]$.

In[16]:=
```
BasisOperators[]
```

Out[16]= $\left\{I_1^- \bullet I_2^-,\ \sqrt{2}\,(I_1^- \bullet I_{2z}),\ \sqrt{2}\,(I_{1z} \bullet I_2^-),\ \frac{I_1^-}{\sqrt{2}},\ \frac{I_2^-}{\sqrt{2}},\ I_1^- \bullet I_2^+,\ I_1^+ \bullet I_2^-,\ 2\,(I_{1z} \bullet I_{2z}),\ I_{1z},\ I_{2z},\ \frac{\mathbb{1}}{2},\ \sqrt{2}\,(I_1^+ \bullet I_{2z}),\ \sqrt{2}\,(I_{1z} \bullet I_2^+),\ \frac{I_1^+}{\sqrt{2}},\ \frac{I_2^+}{\sqrt{2}},\ I_1^+ \bullet I_2^+\right\}$

In[17]:=
```
CoherenceOrder /@ BasisOperators[]
```

Out[17]= {-2, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2}

In[18]:=
```
SpinProductRank /@ BasisOperators[]
```

Out[18]= {2, 2, 2, 1, 1, 2, 2, 2, 1, 1, 0, 2, 2, 1, 1, 2}

basis transformation matrix

In[19]:=
```
OperatorBasisTransformationMatrix[
  ShiftAndZOperatorBasis[],
  CartesianProductOperatorBasis[]
 ] // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & \frac{i}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & \frac{i}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & 0 & -\frac{i}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & 0 & -\frac{i}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0
\end{pmatrix}$$