

## Functions of proof: a comparative analysis of French and Japanese national curricula and textbooks

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## Introduction

- My interests and concerns
  - Diversity of proof and proving in lower secondary schools of different countries;
  - Understand the natures of proof and proving in different countries;
  - Understand the mechanism how they are formed.

Understand the  
difficulty of teaching  
and learning of proof

Ecological perspective  
(Chevallard, 1994, 2002)

Textbook: a kind of data  
to be analysed.

## Introduction

- In my previous work (Miyakawa, 2012)
  - A comparative analysis of French and Japanese textbooks.
  - Different natures of proof in geometry:
    - ✓ Form of proof;
    - ✓ statement to be proven;
    - ✓ properties (theorem, definition, etc.) used in proving.
  - Some functions of proof.
- In this study
  - Further analyse functions of proof in each country;
  - Identify how functions are related to the nature of proof.

## Methodology: a comparative analysis

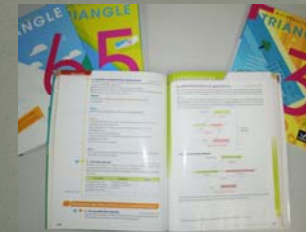
1. Identify functions of proof
  - Analyse **national curricula** and **textbooks** in low. sec.;
    - ✓ Viewpoint: verification, explanation, systematization, discovery, and communication (de Villiers, 1990);
  - Clarify the meaning of identified functions, especially the one which is a **rationale** (raison d'être) why prove.
2. Identify how functions are related to the nature of proof
  - Analyse the natures of related objects and their relations to functions:
    - ✓ Problems: diagrams, statements to be proven;
    - ✓ System of geometry.

## Educational systems and textbooks

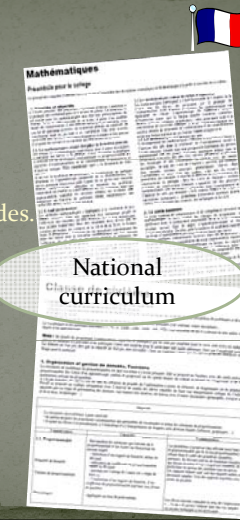
	France	Japan
School	<ul style="list-style-type: none"> <li>Elementary: 5 yrs</li> <li>Lower sec.: 4 yrs</li> <li>Upper sec.: 3 yrs</li> </ul>	<ul style="list-style-type: none"> <li>Elementary: 6 yrs</li> <li>Lower sec.: 3 yrs</li> <li>Upper sec.: 3 yrs</li> </ul>
Textbook	<ul style="list-style-type: none"> <li>Based on <b>national curricula</b></li> <li>Private publisher</li> <li>No approval</li> </ul>	<ul style="list-style-type: none"> <li>Based on <b>national curricula</b></li> <li>Private publisher</li> <li>Approval required</li> <li>Obligation of the use</li> </ul>
Proof	<ul style="list-style-type: none"> <li>Mainly in <b>geometry</b>, and some in algebra</li> <li>Gradually introduced since grade 6, formally in grade 8</li> </ul>	<ul style="list-style-type: none"> <li>Mainly in <b>geometry</b>, and some in algebra</li> <li>Introduced in grade 8</li> </ul>

## Data to be analysed

- France
  - National curriculum;
  - Textbook: *Triangle* (Hatier) and *Sésamath* (Génération 5), guides.



Textbooks: *Triangle* series (Hatier)

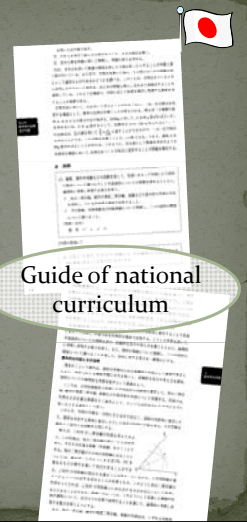


## Data to be analysed

- Japan
  - Guide of national curriculum;
  - Textbook: *New mathematics* series (Tokyo-Shoseki) and guides.

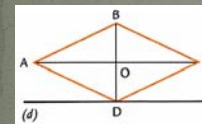


Textbooks: Tokyo-Shoseki series



## "Proof" in French textbook

- Mathematical proof (démonstration)



Prove  
 $(d) \perp (BD)$


**SOLUTION**  
On sait que ABCD est un losange.  
Si un quadrilatère est un losange alors ses diagonales sont perpendiculaires et se coupent en leur milieu.  
Donc (AC) et (BD) sont perpendiculaires.  
On sait que (AC) et (BD) sont perpendiculaires et que (d) et (AC) sont parallèles. Si deux droites sont parallèles et qu'une troisième droite est perpendiculaire à l'une alors elle est perpendiculaire à l'autre.  
Donc (d) et (BD) sont perpendiculaires.

Paragraph

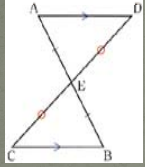
(*Triangle 4e*, 2011, p. 149)

Triadic structure:  
We know that ... (given).  
If ... then ... (property).  
So ... (conclusion).

## “Proof” in Japanese textbook



• Proof (証明)



Prove  
ED = EC

**証明**

△AEDと△BECにおいて

仮定から EA = EB …… ①

対頂角は等しいから

∠AED = ∠BEC …… ②

平行線の錯角は等しいから

∠EAD = ∠EBC …… ③

①, ②, ③より, 1組の辺と2組の角がそれぞれ等しいから △AED ≡ △BEC

合同な図形の対応する辺は等しいから

ED = EC


Well-organised  
(numbered)

Properties  
not if-then form

Semi-paragraph

(Tokyo-Shoseki 2, 2012, p. 110)

## Functions in France



- Prove to **be convinced** or **convince** someone. It is to answer the question “Is it true?”


- Prove to **understand**. It is to answer the question “Why it is true?” (Triangle 4e teacher’s guide, 2011, p. 10)

Communication

Verification  
Explanation

The editing and finalizing of a proof should [...] be presented in a **convincing** way in both oral and written to **communicate** reasoning (MEN, 2008, p. 11)

## Functions in Japan




“**Verifying** by deduction a conjecture obtained by induction or analogy deepens students’ **understanding** on the contents and helps **correlating** and **systematising** the knowledge” (MEXT, 2008, p. 29)

Verification  
Explanation  
Systematisation  
Communication

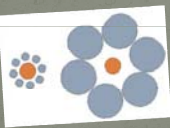
“Teacher should emphasize the importance of **explaining** someone in a **convincing** manner what the student is convinced [...]” (MEXT, 2008, p. 97)

## Justification: Rationale of proof in France




• Justification without perception

- “One cannot prove that a geometrical statement is true by uniquely doing affirmations on a drawing or measurements. [...] In order to prove that geometrical statements are true, one has to carry out mathematical proofs” (Triangle 4e, 2011, p. 147)




Doubt what you see  
(Ebbinghaus illusion)



Doubt what does not exist  
(Kanizsa's Triangle)

Why prove?



(Sésamath 4e, 2011, pp. 218-219)

## Justification: Rationale of proof in France

- Justification without perception
  - From **perceptive** identification of figures to their characterization by **properties** (MEN, 2008)
  - Change of status of geometrical objects: from **physical** objects to **ideal** objects (*Triangle 4e* teacher's guide, 2011)

Proof is a justification tool in theoretical geometry

## Justification: Rationale of proof in Japan

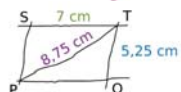
- Justification without exception (general case)
  - “proof is a means to show that a proposition is true **without exception**” (MEXT, 2008, p. 96)
  - “One cannot check out **all triangles** by means of experiments or measurements, but one can show that the sum of interior angles of **any** triangle has 180 degrees by means of a proof like the one above” (Tokyo-shoseki, 2012, p. 98)

Proof is a justification tool for general proposition

## Problems in textbooks


### 42. Du parallélogramme au rectangle

On considère le parallélogramme STOP ci-contre dessiné à main levée.

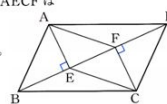


Démontrez que le parallélogramme STOP est un rectangle.

(Sésamath 4e, 2011, p. 146)

Prove rectangle STOP 

問5 □ABCDの頂点A, Cから対角線BDに垂線をひき、対角線との交点をそれぞれE, Fとすれば、四角形AECFは平行四辺形になります。このことを証明しなさい。

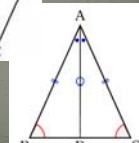
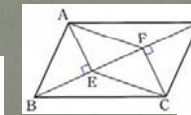
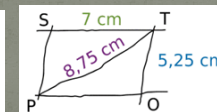
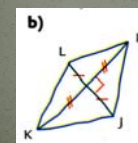


(Tokyo-Shoseki 2, 2012, p. 137)

Prove parallelogram AECF

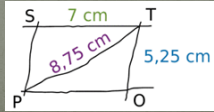
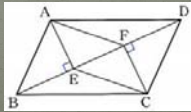
## Diagrams in textbooks

- France
  - “correct” or “incorrect” diagram;
  - a representation (physical object) of an ideal object (*Triangle 4e* guide, 2011, p. 9).
- Japan
  - “correct” diagram;
  - a representative of all figures (MEXT, 2008, p. 96).



**Statements to be proven**

- France
  - Statement about a figure with/without a fixed dimension.
- Japan
  - Statement about a figure without fixed dimension;
  - Justification about a figure with fixed dimension is not “proof”, but just “explanation”;

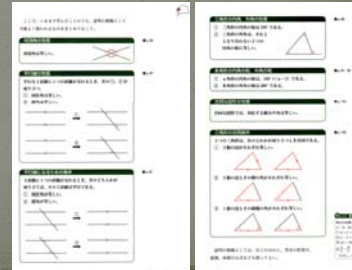



**System of geometry**

Definition, theorem, etc.

- Available properties for proving:
  - Properties admitted at the beginning in Grade 8;
  - Properties proven.
  - Some “axioms” or “postulates” (Tokyo-Shoseki, 2012, pp. 111-112) without these terms.
- As a system
  - Similar to the geometry of *Euclid's elements*.

Quasi-axiomatic geometry



**System of geometry**

- Available properties for proving:
  - Formerly introduced (admitted or justified) in low. sec. school, even in Grade 6;
  - Hard to identify implicit “axioms” or “postulates”.
- As a system
  - Far from the geometry of *Euclid's elements*;

Create network between objects (cf. Miyakawa, 2012)

1. Démontrer que deux droites sont parallèles

Méthode Utiliser

- La propriété des droites parallèles à une même troisième
- La propriété des droites perpendiculaires à une même troisième
- La propriété des côtés opposés d'un parallélogramme, d'un rectangle, d'un losange, d'un carré
- La propriété de la droite qui passe par le milieu de deux côtés d'un triangle
- La propriété de la symétrie centrale
- La propriété des angles alternes-internes ou correspondants

Locally organised-theoretical geometry

**Summary**

- Justification (verification) function
  - France: justification without perception;
  - Japan: justification without exception.
- Problems in textbooks
  - Diagrams;
  - Statements to be proven.
- Systems of geometry
  - France: local organised-theoretical geometry;
  - Japan: quasi-axiomatic geometry.

Rationales why prove.

Two different functions: organisation and systematisation

## For textbook development...

- Close relationship between the choices in the textbook and the functions of proof;
- Provide alternative approach to teach proving and geometry.