A historical perspective on mathematics textbooks

- Approaches to mathematics education change and evolve *e.g., Howson, 1982; Walmsley, 2007*
- Mathematics textbooks can provide a view of approaches of the past *Howson, 2013*
- Thus, mathematics textbooks can bring into sight historical roots of approaches of the present
- Which can lead to a better understanding of approaches in contemporary mathematics textbooks

Mathematics textbook use in Dutch primary schools

- About 94 % of Dutch primary school teachers indicate that a textbook is the main source of their teaching and for about 62 % a textbook is the only source they use *Meelissen et al., 2012; Scheltens et al., 2013*
- At least 80 % of the teachers follow the textbook content for more than 90 % *Hop, 2012*
- Thus, mathematics textbooks are of decisive influence on the Dutch' enacted curriculum

Primary school mathematics textbooks in the Netherlands

- There are currently 7 mathematics textbook series available on the market
- Most of these textbook series are more or less influenced by the Dutch 'Realistic Mathematics Education' (RME) approach
- On approximately 90-95% of the schools, a RME-based textbook series is being used

Realistic Mathematics Education (RME)

- The Dutch RME reform dates from the 1970s (it became known as 'RME' since the 1980s)
- It was a reaction to
  - the then prevailing mechanistic approach in Dutch mathematics education
  - the international 'New Math' reform
- RME aims to give children a better basis for understanding mathematics
- Its name refers to '*zich realiseren*' (to imagine)

e.g., De Jong, 1986; Van den Heuvel-Panhuizen, 2001

RME in Dutch mathematics textbooks

Some characteristics of RME that can be found in textbooks from the 1980s onward are:

- the use of context problems as a source for learning mathematics
- the use of models
- the use of different calculation methods
- the opportunity offered to children to come up with 'own productions', such as self-constructed problems

De Jong, 1986; Van den Heuvel-Panhuizen, 2001

Aim of our study

Get a view of the development of RME-related characteristics in textbooks over the years

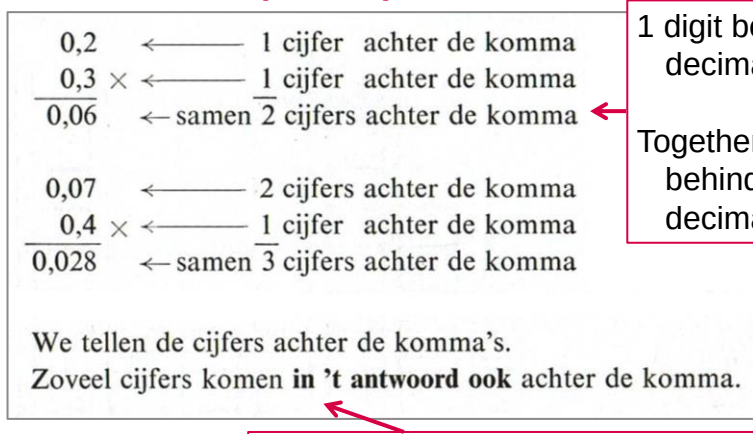
Research question

Were RME characteristics present in a contemporary textbook series, already present in pre-RME textbook series, and if so in what way?

Mathematical focus

Decimal numbers

Decimal numbers in a mechanistic textbook (1978)



1 digit behind the decimal point

Together two digits behind the decimal point

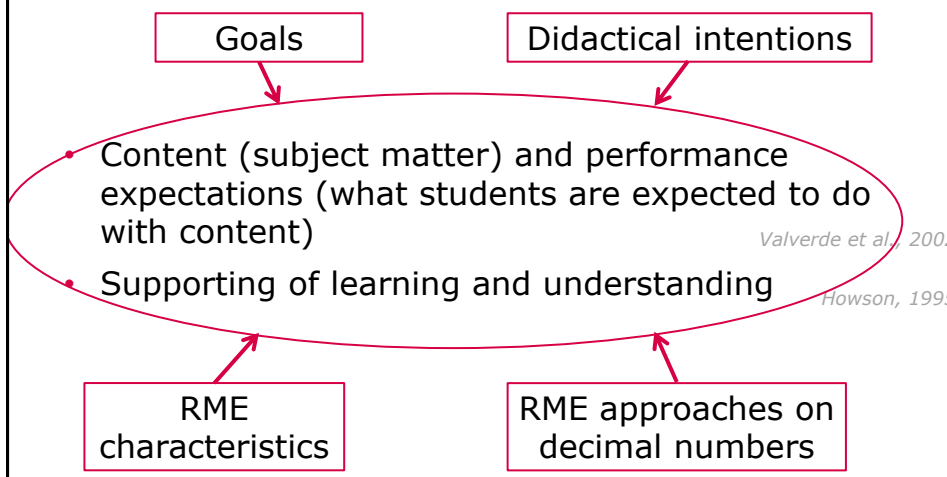
We count the digits behind the decimal points. The answer also gets that much digits behind the decimal points.

RME approaches on decimal numbers

- Interpreting decimal numbers as measurement numbers
- Putting emphasis om estimation with decimal numbers
- Making use of models, especially the number line

De Jong, 1986; Treffers, Streefland & De Moor, 1996

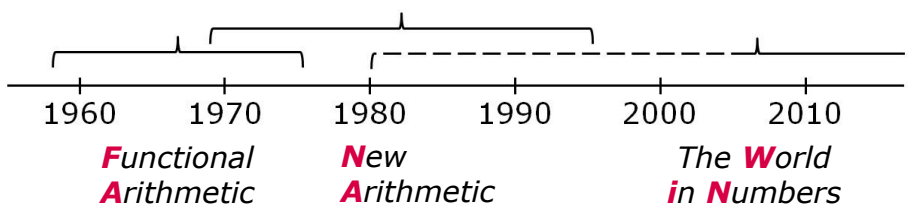
Method Development of analysis framework



Method Analysis framework

Perspective	Specification of the perspective
Format of tasks	Tasks with bare numbers Tasks with measurement numbers Tasks with money
Type of calculation	Tasks on mental calculation Tasks on estimation Tasks on written calculation Tasks on using a calculator
Learning facilitators	Use of contexts as a source for learning Use of the number line as a model Use of other models Offering different calculation methods Offering the opportunity for 'own productions'

Method Textbook series included in the analysis



Method Textbook materials included in the analysis

- Materials for grades 4, 5 and 6
- All materials that are meant for all students and the corresponding directions in the teacher guidelines
- Optional exercises meant for evaluation, repetition or enrichment were left out of our analysis

Method Unit of analysis

Reken uit.

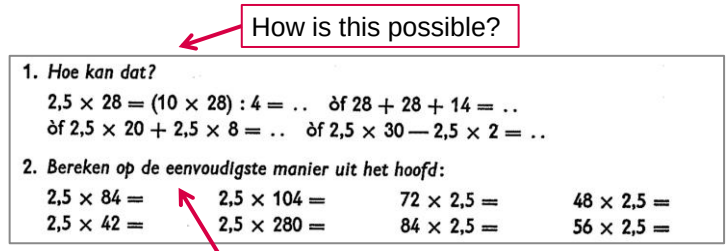
a In de voordeelbak liggen nog een paar coupons grastapijt. Stukken van 12,5 m en 3 m en 7,75 m. Hoeveel meter in totaal?

b 0,5 + 0, 44 =
5,03 + 99 =
0,06 + 60,1 =
7,5 + 3,75 =

c 2,55 + 35 + 102 =
60 + 4,89 + 3,01 =
5,49 + 3,21 + 130 =
12,4 + 0,45 + 3,05 =

- A task is the smallest unit that requires an answer
- A task is often part of a set of tasks
- We considered corresponding directions in the teacher guidelines as belonging to the (sets of) tasks

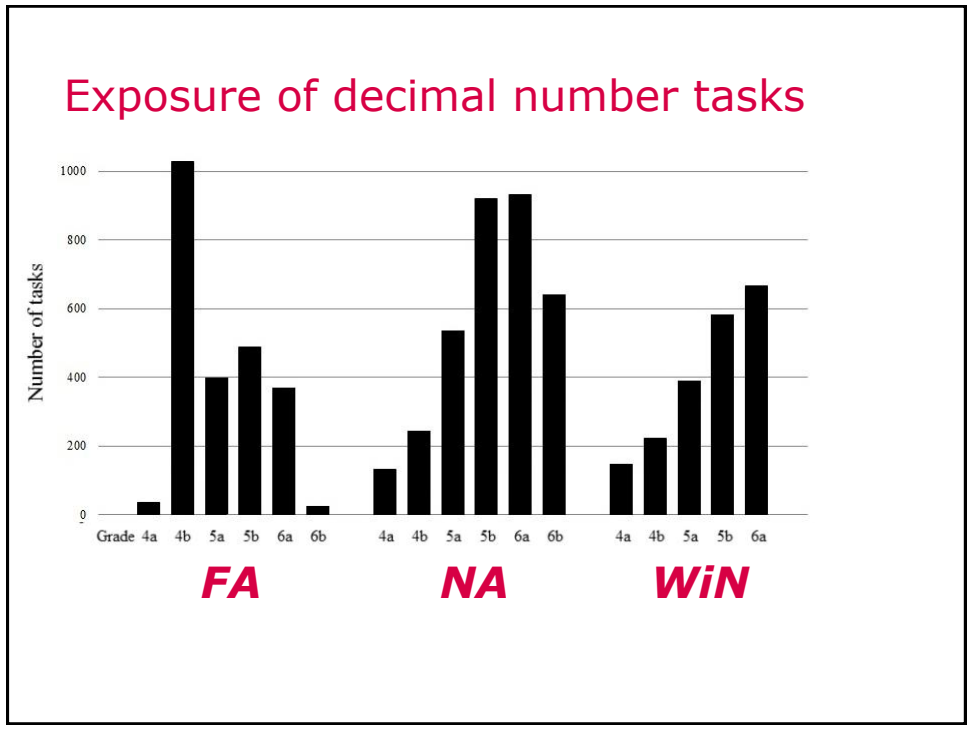
Method Analysis procedure

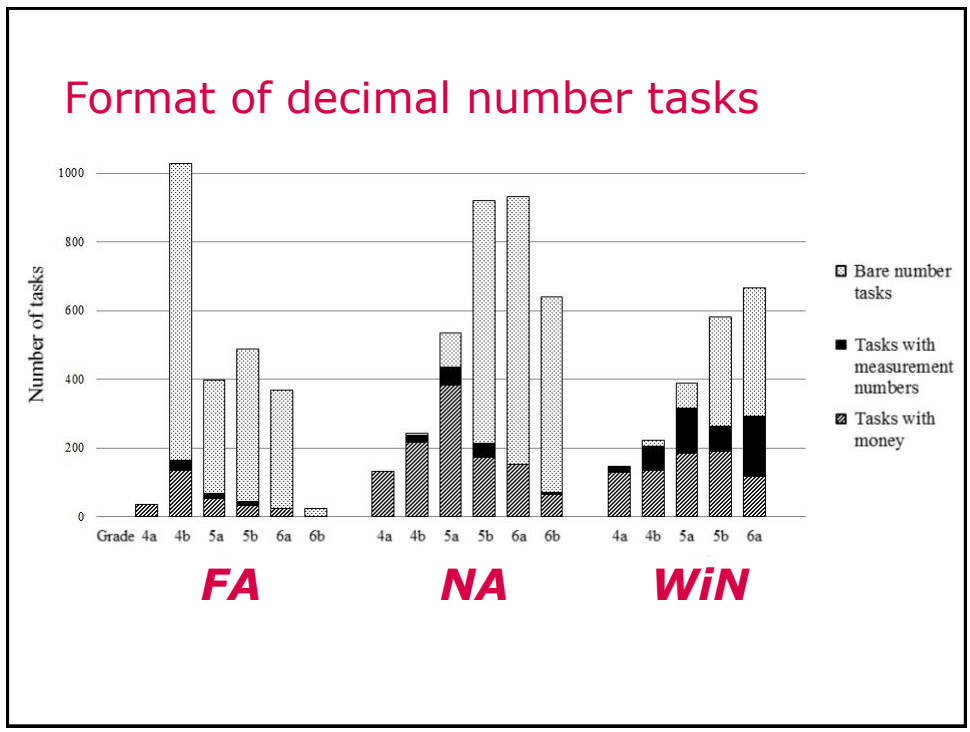


- Format of tasks: Tasks with bare numbers (9 tasks)
- Type of calculation: Mental calculation (9 tasks)
- Learning facilitators: Offering different calculation methods (1 task)

Results

- Exposure of decimal number tasks of different formats
- Measurement numbers and place value
- First exposure of bare decimal numbers
- Types of calculation with decimal numbers, especially estimation
- Use of the number line


Exposure of decimal number tasks
Number of tasks
1000
800
600
400
200
0
Grade 4a 4b 5a 5b 6a 6b 4a 4b 5a 5b 6a 6b 4a 4b 5a 5b 6a
FA
NA
WiN


Format of decimal number tasks
Number of tasks
1000
800
600
400
200
0
Grade 4a 4b 5a 5b 6a 6b 4a 4b 5a 5b 6a 6b 4a 4b 5a 5b 6a
FA
NA
WiN
Bare number tasks
Tasks with measurement numbers
Tasks with money

Tasks on place value with measurement numbers: ***FA***

8,64 gld. De 8 = ...	8,64 m. De 8 = ...
8,64 l. De 8 = ...	8,64 g. De 8 = ...
35,89 m. Deze 35 = ...	ƒ 35,89 Deze 35 = ...
35,89 kg. Deze 35 = ...	35,89 Deze 35 = ...

4286,753 m = 4.. + 2.. + 8.. + 6.. + 7.. + 5.. + 3..
465,93 l = 4.. + 6.. + 5.. + 9.. + 3..
74,9638 hm = 7.. + 4.. + 9.. + 6.. + 3.. + 8..
3,5794 kg = 3.. + 5.. + 7.. + 9.. + 4..

Tasks on place value with measurement numbers: ***NA***

gld	dubb	c
4	7	5

ƒ 8,75 = 8 .. + 7 .. + 5 ..
ƒ 10,36 = 10 .. + 3 .. + 6 ..

m	dm	cm
3	8	4

5,76 m = 5 .. + 7 .. + 6 ..
2,43 m = 2 .. + 4 .. + 3 ..

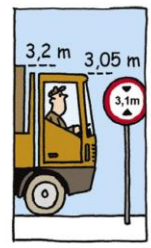
3,75 m = .. m + .. dm + .. cm
Doe ook zo met: 4,25 m = 9,13 m = 7,36 m =
7,86 m = 8,41 m = 4,75 m =

Tasks on place value with measurement numbers: ***WiN***

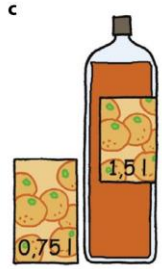
Zet de kommagetallen in de positieschema's.

a

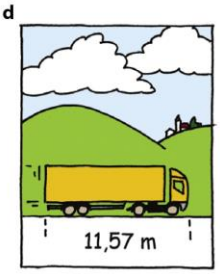
b



c



d



H	T	E	t	h

E	t	h
m	dm	cm

E	t	h
l	dl	cl

T	E	t	h
dam	m	dm	cm

Write the decimal numbers in the place value chart

Tasks on place value with measurement numbers: ***WiN***

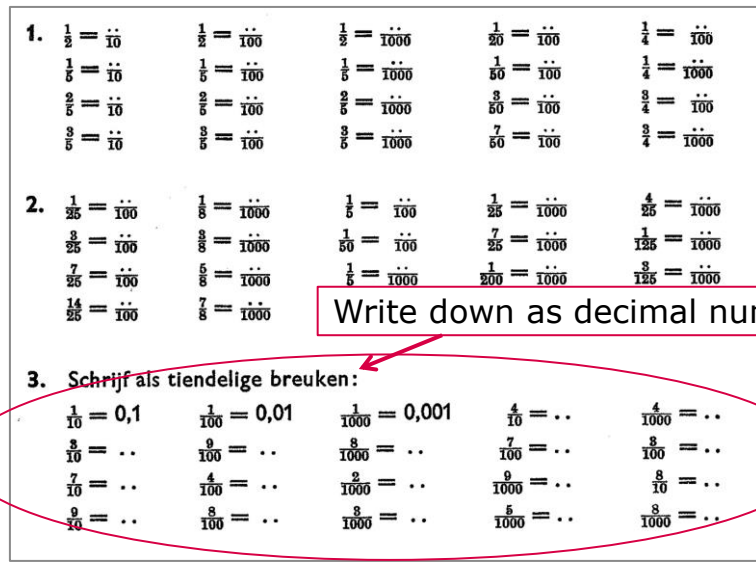
	m	dm	cm	mm
0,225 m				
26,01 m				
7,352 m				
13,057 m				
0,905 m				

km	hm	dam	m	
27	8	0	2	..... km
1	9	5	3	..... km
7	0	8	5	..... km
0	2	7	9	..... km
0	0	5	0	..... km

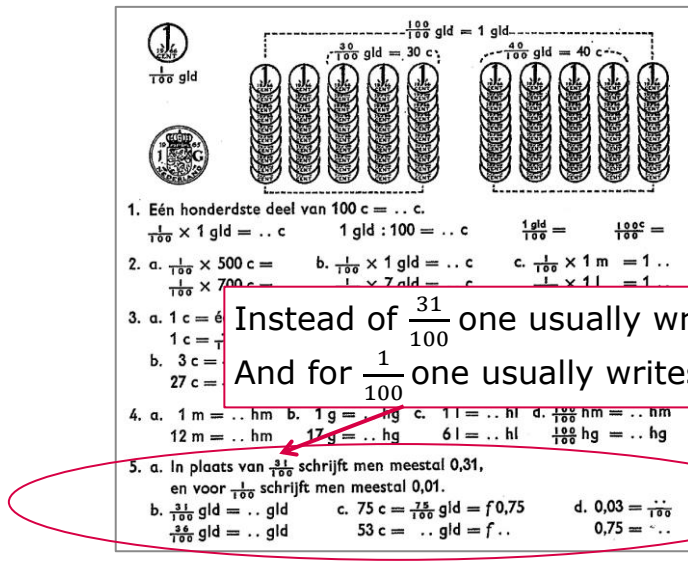
First exposure of bare decimal numbers

	FA	***NA***	***WiN***
When	Grade 4 (2nd half)	Grade 4 (2nd half)	Grade 4 (2nd half)
How	Another way for writing down decimal fractions	Another way for writing down decimal fractions Illustrated with money	Another type of numbers Illustrated with measurement numbers

First exposure of bare decimal numbers: ***FA***



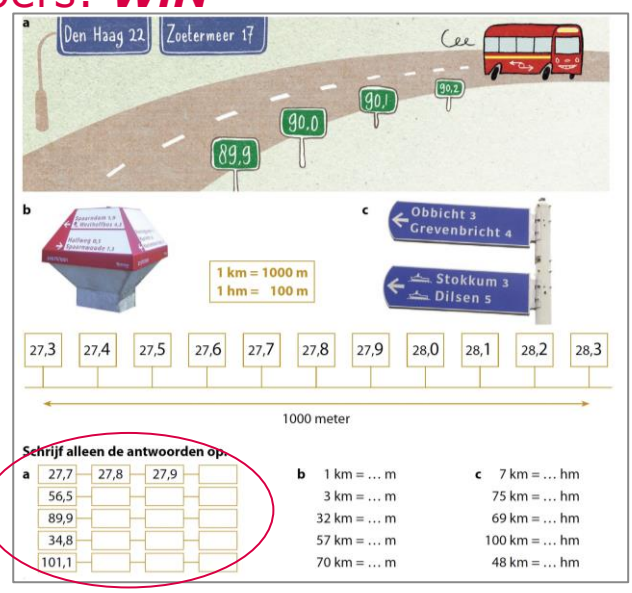
First exposure of bare decimal numbers: ***NA***



1. Eén honderdste deel van 100 c = .. c.
   $\frac{1}{100}$ × 1 gld = .. c 1 gld : 100 = .. c $\frac{1}{100}$ gld = $\frac{1}{100}$ c =
2. a. $\frac{1}{100}$ × 500 c = b. $\frac{1}{100}$ × 1 gld = .. c c. $\frac{1}{100}$ × 1 m = 1 ..
   $\frac{1}{100}$ × 700 c = × 7 gld = c × 1 l = 1 ..
3. a. 1 c = é
   1 c = $\frac{1}{}$
   b. 3 c =
   27 c =
4. a. 1 m = .. hm b. 1 g = .. hg c. 1 l = .. hl d. $\frac{100}{100}$ hm = .. hm
   12 m = .. hm 17 g = .. hg 6 l = .. hl $\frac{100}{100}$ hg = .. hg
5. a. In plaats van $\frac{31}{100}$ schrijft men meestal 0,31,
   en voor $\frac{1}{100}$ schrijft men meestal 0,01.
   b. $\frac{31}{100}$ gld = .. gld c. 75 c = $\frac{75}{100}$ gld = f 0,75 d. 0,03 = $\frac{..}{100}$
   $\frac{36}{100}$ gld = .. gld 53 c = .. gld = f .. 0,75 = $\frac{..}{..}$

Instead of $\frac{31}{100}$ one usually writes 0.31.
And for $\frac{1}{100}$ one usually writes 0.01.

First exposure of bare decimal numbers: ***WiN***



Types of calculation

	FA		NA		WiN	
Mental arithmetic	474	36%	346	24%	321	32%
Estimation	188	14%	472	32%	367	36%
Written arithmetic	671	50%	651	44%	238	24%
Using a calculator					81	8%
	1333	100%	1469	100%	1007	100%

Estimation

	FA		NA		WiN	
Estimation in order to check precise calculation	173	92%	310	66%	112	31%
Estimation as a task on its own	15	8%	162	34%	255	69%
	188	100%	472	100%	367	100%

Estimation: ***FA***

Is more than …, but less than …

1,8 is meer dan .., maar minder dan ..

Is larger than … and smaller than …
Fill in whole numbers

De uitkomst van 2,4 × 7,6 is groter dan .. en kleiner dan ..
Gehele getallen invullen.

Estimation: ***FA***

Write down, or leave blank, without actually calculating: 'wrong', 'too big', 'too small'

Schrijf achter de volgende sommetjes „fout" èn „te groot" of „te klein", als je direct ziet, dat de uitkomst fout is. Niet eerst zelf uitrekenen! Als je meent, dat het antwoord goed kàn zijn, dan zet je er niets achter.

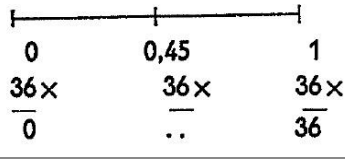
0,95 × 2,357 = 22,3915	40,434 : 8,79 = 4,6
4,82 × 25,94 = 125,0308	1,7472 : 364 = 48
5,267 × 40,3 = 21,22601	17,328 : 4,56 = 38.

Estimation: ***NA***

Do you see it? 5 x *f* 4.25 is less than *f* 25 but more then *f* 20

f 3,87 is meer dan *f* 3 *f* 3,87 is minder dan *f* 4

Zie jij dat ook? 5 × *f* 4,25 is minder dan *f* 25, maar meer dan *f* 20.
Doe ook zo met:
6 × *f* 8,67 3 × *f* 3,57 10 × *f* 2,75 6 × *f* 6,59 9 × *f* 4,38



Waarom zal de uitkomst van 36 × *f* 0,45 minder zijn dan *f* 36?

Why is the answer to 36 x *f* 0.45 less than *f* 36?

Estimation: ***NA***

about

almost

190,65 : 31 → ongeveer 6 679,2 : 14 – bijna 50
145,48 : 12 → ruim 12
Doe ook zo met:
180,75 : 25 = 51,84 : 16 = 47,88 : 28 =
112,8 : 12 = 603,75 : 35 = 598,56 : 48 =

over

Estimation: ***WiN***

Schat het antwoord.

$4 \times €\ 19{,}85 \approx €\ \ldots$

$3 \times €\ 24{,}95 \approx €\ \ldots$

Estimate the answer

What is the correct answer? How do you calculate?

Wat is het goede antwoord? Hoe reken je?

	450			22,5
$30 \times 0{,}15 =$	4,5		$500 \times 0{,}45 =$	0,225
	0,45			225

Estimation: ***WiN***

Bram has made some mistakes with his calculator. What wrong decimal numbers did he enter?

Bram heeft een paar kommafouten gemaakt met de rekenmachine.
Welke foute getallen toetste hij in?

som:	goed of fout?	Bram toetste in:	som:	goed of fout?	Bram toetste in:
24,75 + 37,2 = 396,75			16 × 3,25 = 52		
265,2 + 0,16 = 281,2			8,5 × 16 = 136		
34,75 + 18,85 = 53,6			0,125 × 120 = 0,015		

Schatten. Tusssen welke kommagetallen ligt het antwoord?

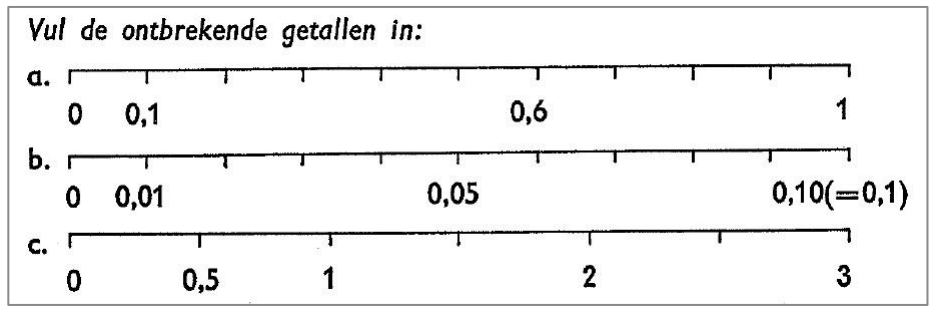
Estimate

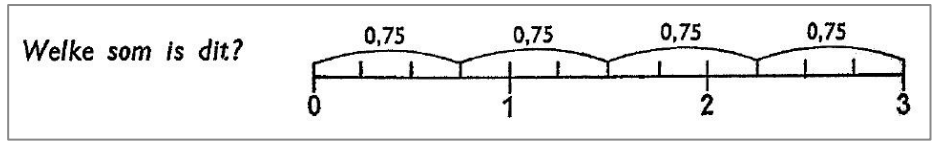
	13,0 en 13,5			14,0 en 14,5
205 : 15 =			880 : 60 =	
	13,5 en 14,0			14,5 en 15,0

Sets of tasks that offer the number line

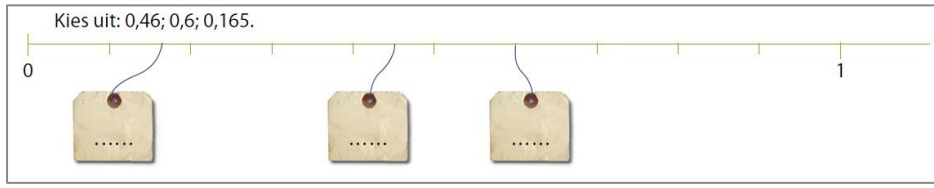
	FA	NA	WiN
Ordering and positioning decimal numbers on the number line	0	2	10
Connecting decimal numbers to equivalent fractions and vice versa	0	0	9
Zooming in	0	0	2
Supporting calculations	0	3	1
	0	5	22

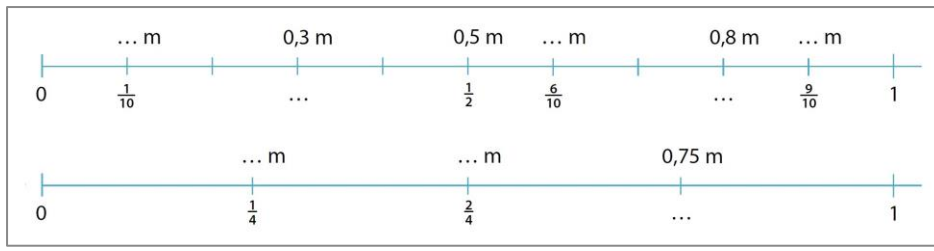
Use of the number line: ***NA***



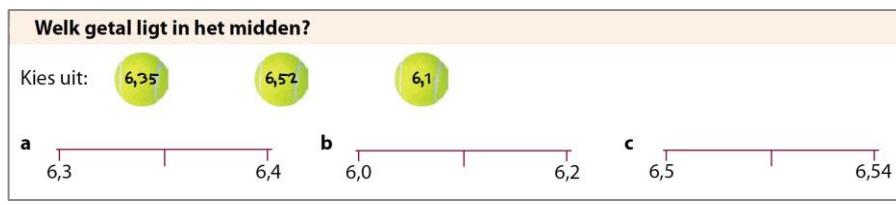


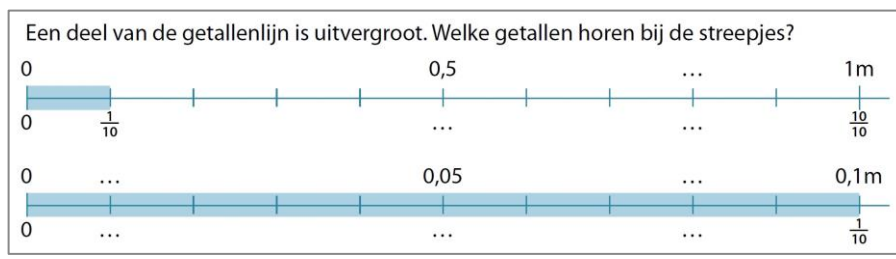
Use of the number line: ***WiN***





Use of the number line: ***WiN***





Conclusions

- Onsets of RME-characteristics present in ***WiN*** were already present in ***FA*** and ***NA***
- The degree of (onsets on) RME-characteristics in ***WiN*** is greater than in ***FA*** and ***NA***
- In some cases the onsets of RME-characteristics in ***FA*** and ***NA*** differ considerably from the RM-characteristics in ***WiN***
- In some cases they are quite similar or even show a certain evolvement

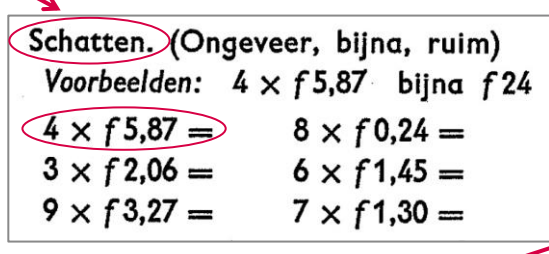
The RME-reform was built at least in part on approaches that were in potency already present in textbooks of the 1960s and 1970s.

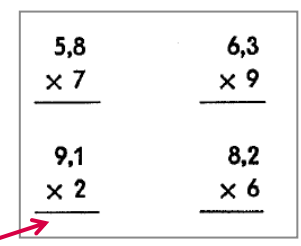
What have we learned? (among other things)

Regarding *method*:

One unit of analysis is not sufficient. The perspectives content, performance expectations and learning facilitators require different units of analysis

Estimate





Teacher guidelines: Estimate, before precise calculation

What have we learned? (among other things)

Regarding *results*:

in the analyzed pre-RME textbook series, we found examples of types of tasks that might form a worthwhile addition to contemporary Dutch textbooks

Is larger than … and smaller than …
Fill in whole numbers

De uitkomst van 2,4 × 7,6 is groter dan .. en kleiner dan ..
Gehele getallen invullen.

What have we learned? (among other things)

```
  5,6
×9,2
-----
 11 2
504 0
-----
515,2
```

Someone had to calculate 9.2 x 5.6.
What mistake did he make?
What is the actual answer?
How big is the difference between the answer alongside and the actual answer?

Bram heeft een paar kommafouten gemaa
Welke foute getallen toetste hij in?

Bram has made some mistakes with his calculator. What wrong decimal numbers did he enter?

som:	goed of fout?	Bra toetste in:	som:	fout?	toetste in:
24,75 + 37,2 = 396,75			16 × 3,25 = 52		
265,2 + 0,16 = 281,2			8,5 × 16 = 136		
34,75 + 18,85 = 53,6			0,125 × 120 = 0,015		

Thank you

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