Back to the past ………
Messages conveyed in textbooks:
A study of mathematics textbooks during the Cultural Revolution in China

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ICMT2014, Southampton
31 July 2014

Introduction

- Mathematics textbooks are supposed to be conveying mathematics knowledge or messages
- At the same time, textbooks are socio-cultural products
- They tend to convey the values of the dominant culture, and there may be a lot of politics involved in the production and adoption of textbooks
- When talking about values, are mathematics textbooks exempted from these cultural and political influences?

Mathematics textbooks

- Mathematics is sometimes conceived of as absolute truth, with minimum human interference
- Mathematics applications are often contrived, meant to illustrate the mathematics rather than genuine application of mathematics to different socio-cultural contexts in the real world
- So a mathematics textbook in say England should look very similar to a mathematics textbook in Thailand or Botswana
- But is this the case?

Textbooks as socio-cultural product

- Intentionally or unintentionally, explicitly or implicitly, textbooks convey messages other than purely mathematical ones
- Also, in most countries, textbooks are at the same time commercial products, which complicates the way how intended values are conveyed through the textbooks (publishers acting as gate-keeper)
- So very often, intended messages or the shared values of a society are conveyed to students in a subtle manner
- The textbooks used during the time of the Cultural Revolution in China, however, were produced in a very special and extreme period of time in the history of China (and of mankind)
- The country was highly politicized, and textbooks were not produced commercially

- As such, no textbooks have illustrated the explicit conveying of different messages more vividly than those published and used during the time of the Cultural Revolution in China
- In order to study how the accepted socio-cultural and political values are conveyed to school children through textbooks, a selected topic in a set of mathematics textbooks from the Cultural Revolution period in China is analysed and compared to a set of textbooks published in Hong Kong roughly at the same time, and to a set of textbooks currently used in China
- (Another important focus of study is how the textbooks were used in the Cultural Revolution classroom, but this is beyond the scope of this presentation.)

Past Studies

- Jules (1991): "Building Democracy: Content and ideology in Grenadian Educational Texts, 1979-1983"
- Grenadian Revolution in 1979: the textbooks at the time elucidates how textbooks became part of "the hegemonic restructuring of knowledge and ideological consciousness"
- For mathematics, studies on messages conveyed in textbooks were mostly on values of mathematics and education (e.g., Seah and Bishop, 2000)
- Very few on ideological and political messages
- Does this mean that mathematics textbooks do not contain ideological messages?

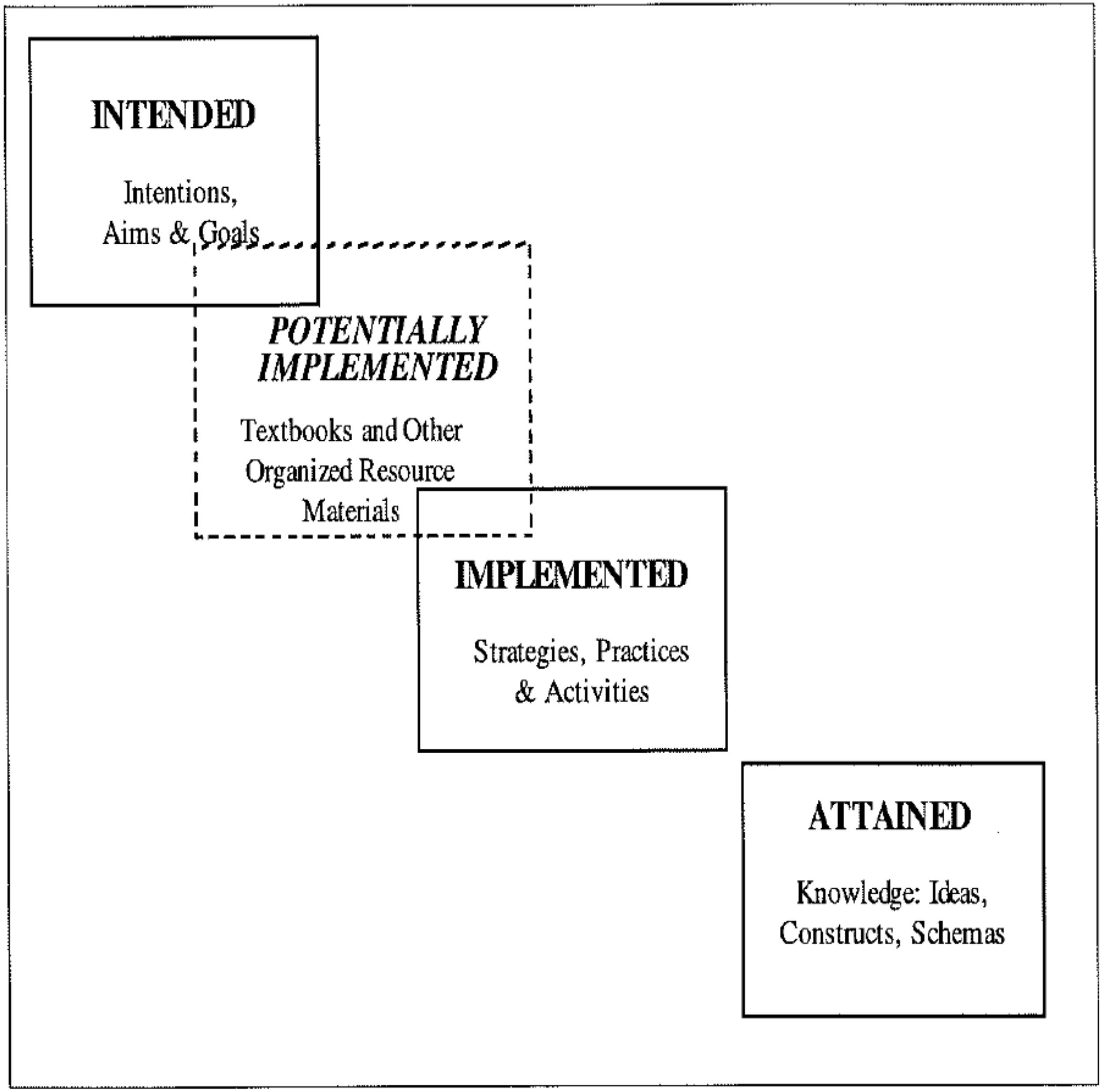
An, Capraro & Hao (2011)

- "Ideological Representations in Chinese Mathematics Textbooks during the Cultural Revolution Decade: A Relational Analysis of Selected Textbooks from 1966-76", *Journal of Curriculum Theorizing*, 27, 3.
- Studied 28 pages from four textbooks and examined the ideological content according to the different presentation (verbal, pictorial, and numerical)
- The content was then related to the political and historical issues of the time
- But the study is not particular on messages conveyed in the textbook

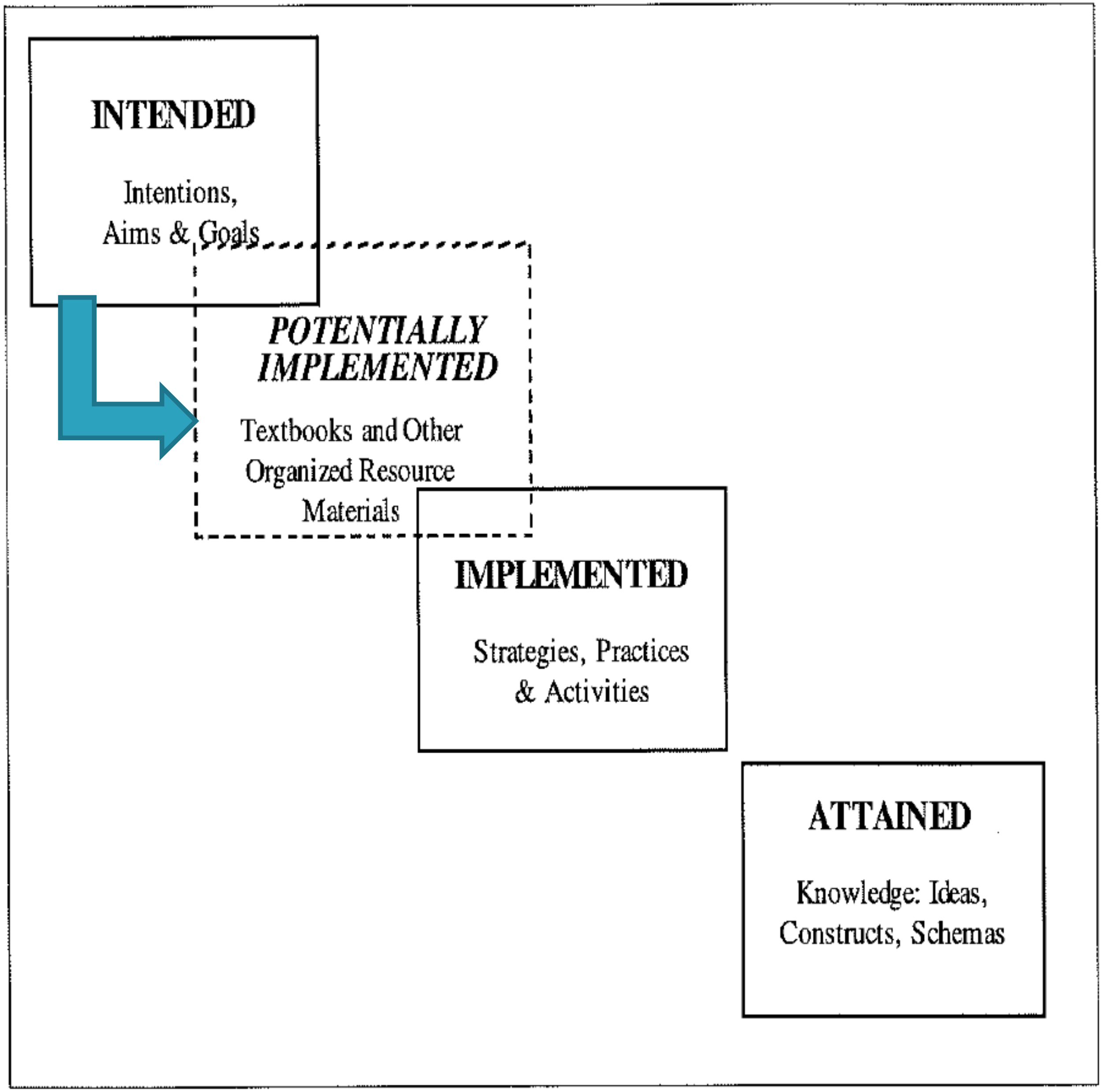
Theoretical Orientation

- In terms of the TIMSS terminology, are textbooks part of the intended curriculum or the implemented curriculum?
- In this presentation, textbooks are conceived of as part of the “potentially implemented curriculum”
- It mediates between the intended and implemented curriculum
- In this presentation, we study how values in the intended curriculum are conveyed through the textbook, which in turn should impact the implemented curriculum
- Critical educator: “whose knowledge, in what form, how it is selected and by whom, and to what ends”

The TIMSS framework



The TIMSS framework



Textbooks as socio-cultural product

- “From a socio-cultural perspective the mathematics textbooks can be regarded as an artifact in the broad sense of the term. It is historically developed, culturally formed, produced for certain ends and used with particular intentions.”(Rezat, 2006)
- Textbooks are “ideological tools to promote a certain belief system and legitimize an established political and social order” (Apple, 1992)
- The Dutch’s promotion of middle class values in the first half of the 20th Century (mentioned by Kilpatrick in the first plenary)
- What shared cultural experience did the textbook provide for Chinese students during the Cultural Revolution which may explain some of their behaviour?
- And what cultural experience does the textbook provide for our students today which may mould their values and behaviour?

The Cultural Revolution in China

- The Great Proletarian Cultural Revolution, commonly known as the Cultural Revolution, is a social-political movement that took place in China roughly between 1966 and 1976
- Time of great political and social turmoil
- Espoused goal: remove capitalist and traditional cultural elements from the country in order to achieve an idealistic form of communism
- Explicit in articulating communist values and criticizing the capitalist and traditional Chinese cultural elements and values
- One important arena for this revolution is education, and this arena provides an excellent opportunity for studying the interplay between the cultural context and educational goals, through examining the messages conveyed in the textbooks

Methodology

Data Source

Textbooks during this period of time are available for the following provinces/cities:

- Beijing
- Hunan
- Shandong
- Shanghai
- Tianjin

Since Beijing is the capital of the country, and the complete set of textbooks from Beijing is available, this set of textbooks is chosen for study

For contrast

- A set of mathematics textbooks published in Hong Kong around the same time period (late 1960s and early 1970s) was used for comparing the messages conveyed in the textbooks in the different socio-political contexts
- This is the time in Hong Kong when the modern mathematics movement dominated
- Another set of mathematics textbooks published in Beijing and currently in use is also used for comparison
- In this presentation, the topic of Directed Numbers and Rational Numbers is selected for discussion

The textbooks chosen for study

Beijing 1972

Shu Xue (Mathematics), Beijing Municipal Department of Education, Beijing: People's Press (Book 1) (for 7th Grade)

Hong Kong 1972

Understanding Mathematics: A New Approach to Mathematics for Secondary Schools, edited by Lee Yee, Hong Kong: Swindon Book Co., 1972 (revised edition) (Book 3) (for 8th Grade)

Beijing 2005

Shu Xue (Mathematics), Beijing: Beijing Normal University Press. Grade 7, Volume One

[Hong Kong 2004

Exploring Mathematics, Leung, F.K.S., Chu, W.M. and Luk, M.L., Hong Kong: Oxford University Press (Book 3) (For 8th Grade)]

	Beijing	Hong Kong
1972	(Cultural Revolution) *Shu Xue*	(Modern Mathematics Movement) *Understanding Mathematics*
2004 /5	(Contemporary) *Shu Xue*	(Contemporary) *Exploring Mathematics*

Data Analysis

- In addition to comparing the physical outlook of the books, we analysed the textbooks as follows:
- The document analysis methodology in TIMSS curriculum analysis was adapted for use in this study
- The book was divided into units, and each unit was then divided into blocks and coded for different block types
- For each type of blocks, the messages conveyed by the block were then coded
- A subset of the blocks was coded by two independent researchers and inter-rater reliability was computed
- Coding of the whole dataset was conducted after an inter-rater reliability of over 85% was achieved

Block types used in TIMSS

1. Narrative
2. Related narrative
3. Unrelated instructional narrative
4. Related graphic
5. Unrelated graphic
6. Exercise/question set
7. Unrelated exercise/question set
8. Activity
9. Worked example
10. Other
11. Missing

Block types used in this study

TIMSS	This study
1. Narrative 2. Related narrative 3. Unrelated instructional narrative	N: Narrative
4. Related graphic 5. Unrelated graphic	G: Graphs and Tables Ph: Photos and pictures
6. Exercise/question set 7. Unrelated exercise/question set	E: Exercise/question set
8. Activity	A: Activity
9. Worked example	W: Worked example
10. Other	O: Others
11. Missing	

Two kinds of blocks: Presentation of ideas and Mathematics problems and activities

Presentation of ideas

- N = Narrative: verbal or textual
- G = Graphs or Tables: mathematical
- Ph = Photos or pictures: iconic representations

Mathematics problems and activities

- W = Worked examples
- A = Activities
- E = Exercise/question set

IV. RESOLUCIÓN DE UN SISTEMA DE ECUACIONES POR MÉTODOS ALGEBRAICOS U.33 T.2 T.1

Para resolver un sistema de dos ecuaciones con dos incógnitas, además del método gráfico que ya vimos podemos emplear otros procedimientos de carácter algebraico. Es importante hacer notar que después de estudiar algunos, tú puedes elegir el que más se te facilite.

1. Método de sustitución

a) Analiza el desarrollo del siguiente problema.

2 Se tienen \$ 500 en monedas de \$ 5 y de \$ 20. Si dieron 55 monedas ¿cuántas son de 5 y cuántas de 20?

Sea x la cantidad de monedas de \$ 5 y m la cantidad de monedas de \$ 20. T.6

5x representa la cantidad de dinero en monedas de \$ 5.

20m representa la cantidad en monedas de \$ 20.

$$5x + 20m = 500$$
$$x + m = 55$$

En el método de sustitución se despeja una de las variables en una de las ecuaciones y se sustituye en la otra, para quedar una ecuación de primer grado con una incógnita. Resolvemos la ecuación y obtenemos el valor de una de las incógnitas. El valor de la otra variable lo obtenemos al sustituir el valor conocido en la ecuación donde quedó despejada la otra incógnita. 3 T.1

$5x + 20m = 500 \ldots$ (1)

$x + m = 55 \ldots$ (2)

En la ecuación (2) despejamos x

$$x + m = 55$$
$$-m = -m$$
$$x = 55 - m$$

queda $x = 55 - m$

En la ecuación (1) sustituimos el valor de x $\quad 5x + 20m = 500$

$$5(55 - m) + 20m = 500$$

Resolvemos la ecuación de primer grado con una incógnita y obtenemos el valor de m

$$275 - 5m + 20m = 500$$
$$275 + 15m = 500$$
$$-275 \quad = -275$$
$$15m = \boxed{}$$
$$m = \frac{\boxed{}}{15}$$
$$\boxed{m = }$$

Si sabemos que $x = 55 - m$, sustituimos m por su valor.

$$x = 55 - 15$$
$$\boxed{x = 40}$$

Comprobación

$5x + 20m = 500 \qquad 5(40) + 20(15) =$ ______ + ______ = ______

$x + m = 55 \qquad 40 + 15 =$ ______

...n 40 monedas de \$ 5 y 15 monedas de \$ 20

b) Veamos otro caso: T.6

4

$4x + 3y = 5 \ldots$ (1)

$2x + y = 1 \ldots$ (2)

Despejamos y en la ecuación (2)

$$2x + y = 1$$
$$-2x \quad = -2x$$
$$y = 1 - 2x$$

Sustituimos y en la ecuación (1)

$$4x + 3y = 5$$
$$4x + 3(1 - 2x) = 5$$

Resolvemos la ecuación

$$4x + 3 - 6x = 5$$
$$-2x + 3 = 5$$
$$-3 = -3$$
$$-2x = \underline{}$$
$$x = \frac{}{-2}$$
$$\boxed{x = }$$

Como $y = 1 - 2x$, sustituimos el valor de x

$$y = 1 - 2(-1) = 1 + 2 = 3$$
$$\boxed{y = 3}$$

...omprobación

$4x + 3y = 5 \qquad 4(-1) + 3(3) =$ ______ + ______ = ______

$2x + y = 1 \qquad 2(-1) + 3 =$ ______ + ______ = ______

Messages

Bishop (1988): Rationalism, Objectism, Control, Progress, Openness, Mystery

Seah (2000): mathematical values, mathematics educational values, and general educational values

Pepin et al. (2001):

- Mathematical intentions
- Pedagogical intentions
- Sociological contexts
- Cultural traditions

Messages coded for this study

Messages about:

- Mathematics
- Mathematics learning and teaching
- Political, social and cultural messages

Examples are given below

Messages about Mathematics (Hong Kong, 1972, p.44)

In the study of natural numbers you have learnt that a set of objects called numbers is a number system if it behaves like the set of natural numbers with respect to two operations and their properties. In this chapter you have learnt that the set of rational numbers is closed, commutative, and associative under the operations of addition and multiplication. Moreover, multiplication of rational numbers is distributive over addition. Therefore, R, together with the operations of addition and multiplication and some of their properties, forms a number system.

Message about (mathematics) learning (and teaching) (Beijing, 1972, p.9)

在社会实践中，**就人类认识运动的秩序说来，总是由认识个别的和特殊的事物，逐步地扩大到认识一般的事物。**通过温度高低的比较，使我们进一步认识到：

一个数轴上的点越往右边，它表示的数就越大。

As far as human knowledge of the order of events is concerned, (we) always start with individual and specific things, and progressively expand to knowledge of general things. Through comparing temperatures, we know further that the more to the right are points on a number line, the bigger the number.

Political messages

一九五八年，首都革命人民遵照毛主席关于“水利是农业的命脉”的教导，修建了十三陵水库（图1—8）。在水库施工最紧张的时刻，我们伟大领袖毛主席亲临现场和工地军民一起劳动。毛主席的到来，大大激发了广大军民的劳动热情，仅仅用了160天，水库就建成了。十几年来，十三陵水库在蓄洪和灌溉方面发挥了很大的作用。拦蓄洪水时，水库的水位就上升；灌溉农田时，水位就下降。水库管理人员需要经常观测水库水位的变化。

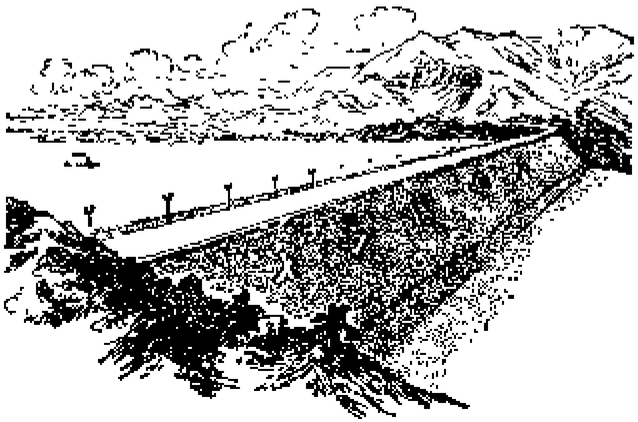


图 1—8

Political messages

In 1958, following the teaching of Chairman Mao that "water works is the lifeline of agriculture", the revolutionary workers in the Capital built the Thirteen Tomb Dam (Figure 1-8). At the most critical time of the construction of the dam, our great leader Chairman Mao came to the construction site himself and worked with soldiers and workers at the site. The presence of Chairman Mao encouraged the working enthusiasm of the soldiers and workers, and the dam was finished in only 160 days ……

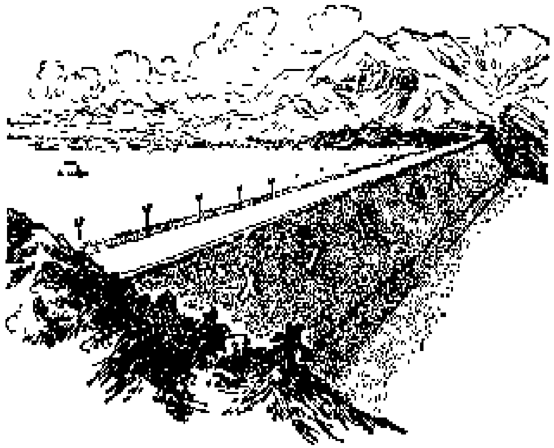


图 1—8

Coding of messages conveyed

Code	Block Types
N-M	Narrative: Mathematics
N-L	Narrative: Learning and Teaching
N-P	Narrative: Political
Non-Narrative Blocks	
E	Exercise (M, L or P)
W	Worked examples (M, L or P)
G	Graphs or Tables (M, L or P)
Ph	Photos or Pictures (M, L or P)
A	Activities (M, L or P)

Coding of Exercise

S = Short questions

L = Long questions (numerical)

A = Application problems

T = True or False questions

W = Questions that require explanation, justification or exploration (Why questions)

P = Questions of Proof

Short questions and long questions (Beijing 2005, p.46)

3. 比较下列每组数的大小：

(1) -10，-7；　　(2) -3.5，1；

(3) $-\frac{1}{2}$，$-\frac{1}{4}$；　　(4) -9，0；

(5) -5，3，-2.7；　　(6) 3.8，-4.1，-3.9.

4. (1) 点 A 在数轴上距原点 3 个单位长度，且位于原点左侧．若将 A 向右移动 4 个单位长度，再向左移动 1 个单位长度，此时 A 点所表示的是什么数？

Short questions and long questions (Beijing 2005, p.46)

3. Compare the magnitudes of the following sets of numbers:

(1) – 10, - 7; (2) – 3.5, 1

(3) – ½, -1/4 (4) -9, 0;

(5) -5, 3, -2.7; (6) 3.8, -4.1, -3.9

4. (1) Point A is 3 units from the original on the number line, and it is to the left of the origin. If we move point A 4 units to the right, and then move it one unit to the left, what number does point A represent now?

Application problems

Express in scientific notation the following:

(1) During the Great Proletarian Cultural Revolution, our great leader Chairman Mao met the Red Guards and other revolutionary masses eight times. The number of people met reached 13,000,000.

(3) During the war of liberation, the great People's Liberation Army destroyed 8,070,000 of Chiang's army

True or False questions

2. Tell whether each sentence below expresses a true statement or a false statement. $U = R$.

(a) $\frac{9}{10} = 0 + \frac{9}{10}$

(b) $0 \cdot (-\frac{5}{3}) = -\frac{5}{3}$

(c) $(-14) + 0 = -14$

(d) $\frac{8}{3} \cdot 1 = 1$

(e) $0 + \frac{7}{3} = \frac{7}{3}$

(f) $1 \cdot (-8) = -8$

(g) $\frac{1}{5} + 0 = \frac{1}{5}$

(h) $\frac{8}{5} + 1 = \frac{8}{5}$

(i) $1 \cdot (-\frac{8}{7}) = -\frac{8}{7}$

Questions that require explanation, justification or exploration

Beijing 2005

- The letter a represents a number, what does $-a$ represent? Must $-a$ be a negative number?
- If the product of two numbers is a negative number, what can you say about the signs of the two numbers? How about when the product of two numbers is positive? Can you generalize it to the product of many numbers?

Questions of proofs

22. Prove that the sum and the product of 2 even integers are even. [Hint: Let 2x and 2y represent two even numbers, where x and y are integers.]

23. Prove that the sum of 2 odd integers in an even number and their product is an odd number. [Hint: Let 2x + 1 and 2y + 1, where x and y are integers, represent two odd integers.]

Results

1. The physical outlook of the book
2. Introduction to the book
    - The introduction to the book is the official statement about the book
    - It sometimes states what messages are intended
3. The first two examples in introducing the topic
    - In the textbook, most mathematical topics are introduced through examples, and the examples chosen convey messages about the nature of the topics concerned
4. Block types and messages conveyed in each block
5. Types of exercise

1. The physical outlook of the book

Beijing 1972

第一章　有　理　数

一　有理数中的一些概念

1. 正数和负数

先看下面的问题：

在万恶的旧社会，放羊娃被迫在零下15°C的冰天雪地里为地主放羊，可是地主却在零上15°C的屋子里过着剥削寄生的生活（图1—1）。这里，零下15°C和零上15°C虽然是同一种量，但是它们的意义是相反的，一个高于零度，一个低于零度。

图　1—1

又如，单人掩体战壕的积土部分，需要高出地面30厘米，而掩体战壕的底，需低于地面120厘米（图1—2）。这里，"高出"地面30厘米和"低于"地面120厘米，也是两个意义相反的量。

"矛盾是普遍地存在着"，在三大革命实践中，存在着大量互相矛盾的、意义相反的量，比如产量的增加与减少，财政的收入与支出，平整土地的挖方量与填方量等等。

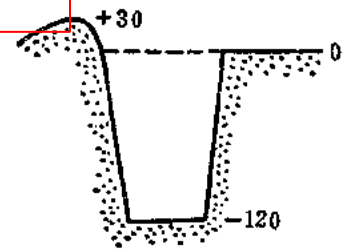


图　1—2

为了区别具有相反意义的量，我们在数的前面分别加上不同的符号。例如，对于温度，以0°C为标准，把零上15°C记作+15°C，零下15°C记作-15°C；同样，对于单人掩体的战壕问题，以地平面为基准（记作"0"），把高出地面30厘米记作+30厘米，低于地面120厘米记作-120厘米。

象+15、+30这样带有"+"（读作"正"）号的数叫做正数，-15、-120这样带有"-"（读作负）号的数叫做负数*。以前我们在算术中学过的数都是正数（零除外，它既不是正数也不是负数）。一般正数前面的"+"号省略不写，如+30就写成30。

例　1　工人师傅加工一根直径为50毫米的轴，规定加工后轴最粗不能比50毫米大0.015毫米；

* 这里的"+"、"-"是用来表示数的性质的，因此叫做性质符号。

最细不能比 50 毫米小 0.01 毫米。试用正负数表示。

解：比 50 毫米大多少毫米和小多少毫米是相反意义的量，用正负数表示如下：

比 50 毫米大 0.015 毫米，记作 +0.015 毫米；

比 50 毫米小 0.01 毫米，记作 -0.01 毫米。

在图纸上一般用 $50^{+0.015}_{-0.01}$ 表示。工人师傅把 +0.015 叫做上偏差，-0.01 叫做下偏差。

"人的正确思想，只能从社会实践中来，只能从社会的生产斗争、阶级斗争和科学实验这三项实践中来。"通过上面例子的分析，我们看出：正数和负数的概念不是凭空从头脑中想出来的，而是实际生活中互相矛盾的、意义相反的量的反映。它们构成了数学中的一对矛盾。矛盾着的两个方面不是孤立地存在的，没有正，也无所谓负；没有负，也无所谓正，它们是对立的统一。

学习了负数以后，我们学过的数就有：

$$
\begin{cases}
\text{整数}\begin{cases}\text{正整数(自然数)} & \text{如 } 1,2,3,\cdots\cdots \\ \text{零} & 0 \\ \text{负整数} & \text{如 } -1,-2,-3,\cdots\cdots\end{cases} \\
\text{分数}\begin{cases}\text{正分数} & \text{如 } 4\frac{2}{3},0.5,\cdots\cdots \\ \text{负分数} & \text{如 } -7\frac{4}{5},-7.9,\cdots\cdots\end{cases}
\end{cases}
$$

整数和分数统称有理数。

从某地分别向东、西方向开出两辆汽车。第一辆向东走 20 公里，我们记作 +20 公里；第二辆向西走 40 公里，我们记作 -40 公里。如果比较两辆汽车所走的距离，我们只考察所走路程的数量而不管它的方向，这时我们把所走的路程就用正数写出来。如第一辆汽车所走的距离，记作 +20；第二辆汽车所走的距离，不记作 -40，而记作 +40。通常，我们把这个正数叫做原来那个数的绝对值。如 +20 的绝对值是 +20；-40 的绝对值是 +40；+3.5 和 -3.5 的绝对值都是 +3.5；对于 0 来说，它的绝对值还是 0。要表示一个数的绝对值，可以在这个数的两旁各画一条竖线。例如，$|+20|=20$，$|-40|=40$，$|+3.5|=3.5$，$|-3.5|=3.5$，$|0|=0$。

例 2 求 -8、-0.5、$+\frac{1}{8}$ 的绝对值。

解： $|-8|=8$；$|-0.5|=0.5$；$\left|+\frac{1}{8}\right|=\frac{1}{8}$。

练 习

1. 读出下列各数：

$+2$，-6，-3，$+0.4$，$+0.75$，-8.2，$+6.3$。

2. 用正数和负数表示下列具有相反意义的量：

Hong Kong 1972

CHAPTER 2

RATIONAL NUMBERS AND SYSTEM OF RATIONAL NUMBERS

1. A REVIEW

Up to now our mathematics lessons have covered mainly two number sets, the set of natural numbers, N and the set of rational numbers of arithmetic, R_a .

The two basic operations of natural numbers are addition and multiplication. Each of these operations is closed, commutative, and associative; and multiplication is distributive over addition. N does not contain negative numbers and fractional numbers. Hence subtraction and division are not defined in N.

The set of rational numbers of arithmetic has all the properties of addition and multiplication of the set of natural numbers. However there is an advantage in operating with rational numbers of arithmetic. Every member in R_a is a fractional number, hence we are free to divide one number by another, except zero. This means division is closed in R_a when zero is discarded.

Every natural number can be expressed as a fractional number. We have no doubt, then, that the set of natural numbers is a subset of the set of rational numbers of arithmetic. Like N, R_a is a number system under addition and multiplication.

2. NECESSITY OF INTRODUCING NEGATIVE RATIONAL NUMBERS

Consider the following question:

(1) If a and b are natural numbers, is there always a natural number x such that $x + a = b$ will express a true statement?

Replace a by 10 and b by 4. Can x be a natural number?

(2) The temperature of a city at 6 a.m. was $6\frac{1}{2}$ °F and at 6 p.m. it dropped by $8\frac{2}{3}$ °F. What was the temperature at 6 p.m.?

In order to give meaning to the answers of the problem above, the set of natural numbers and the set of rational numbers of arithmetic are not sufficient. We need to extend the number system so that it includes negative rational numbers as well.

3. DEFINITION OF RATIONAL NUMBERS

The notion of a rational number is built on the basis of R_a. Every rational number of arithmetic can be expressed as the difference of two rational numbers of arithmetic. Moreover, the expression is not unique. For example, the number $\frac{2}{3}$ is

$$3 - 2\frac{1}{3},\ 1 - \frac{1}{3},\ 2 - 1\frac{1}{3},\ \text{and so on.}$$

Obviously, every rational number is set of ordered pairs of rational numbers of arithmetic. Thus

$$\frac{2}{3} \text{ is } \{(3,\ 2\tfrac{1}{3}),\ (1,\ \tfrac{1}{3}),\ (2,\ 1\tfrac{1}{3}),\ \ldots\}.$$

As a second example,

$\{(2\frac{1}{3},\ 3),\ (\frac{1}{3},\ 1),\ (1\frac{1}{3},\ 2),\ \ldots\}$ is the rational number $\frac{-2}{3}$, which has the opposite sense of $\frac{2}{3}$.

It can be seen that for any two pairs in any one of the sets, (a, b) and (c, d), we have

$$a + d = c + b.$$

To test whether an ordered pair (x, y) is in the special set, say the first set, choose any known element from it as $(1,\ \frac{1}{3})$, and test the truth of

$x + \frac{1}{3} = 1 + y$. If the sentence expresses a true statement, then (x, y) belongs to the set.

All ordered pairs representing the same rational number are called equivalent ordered pairs.

In order that an ordered pair (a, b) may belong to the set R_a, a must be greater than b. But if a is less than b, then it is not a rational number of arithmetic but is a negative rational number as in the second example above. The system so constructed is the extension of the system of rational numbers of arithmetic. Each member in the system is called a rational number. Hence a rational number may be positive, or negative, or zero.

Definition: A rational number is a set of equivalent ordered pairs of rational numbers of arithmetic, where any two pairs (a, b) and (c, d), belong to the

6. 分别找出一个满足下列条件的整数：

(1) 加上 -15，和大于 0；

(2) 加上 -15，和小于 0；

(3) 加上 -15，和等于 0 .

Beijing 2005

问题解决

1. 下面是一页账单，但有一部分破损了，你能根据上面残余的数字算出这一页最后的结余吗？

日期	支出或存入	结余	注释
20040526	− 120.00	9 546.00	
20040612	− 150.00		
20040625	280.00		
20040705	− 315.00		
20040812	− 540.00		
20040906	− 470.00		

联系拓广

*1. 将 -8，-6，-4，-2，0，2，4，6，8 这 9 个数分别填入右图的 9 个空格中，使得每行的 3 个数、每列的 3 个数、斜对角的 3 个数相加均为 0 .

（第 1 题）

5 有理数的减法

下表是《北京青年报》2001 年 4 月 9 日刊登的全国主要城市天气预报.

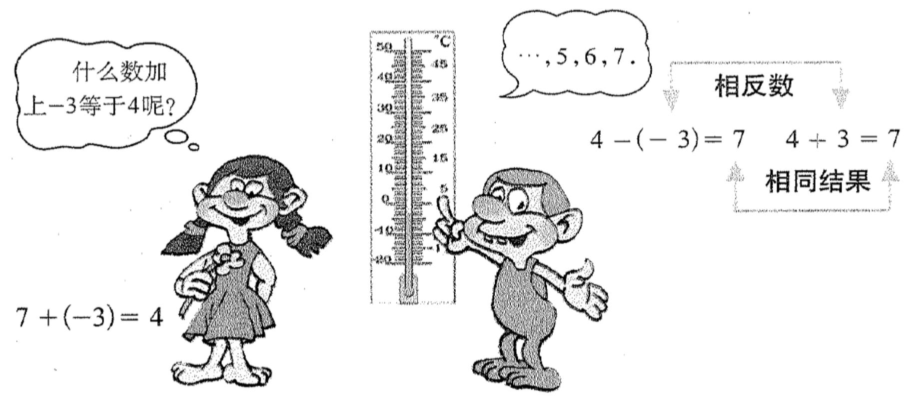
北京专业气象台

全国主要城市天气预报

城市	天气	高温	低温	城市	天气	高温	低温
哈尔滨	小雨	15	6	长春	多云	18	10
沈阳	小雨	19	7	天津	小雨	12	8
呼和浩特	雨夹雪	8	−3	乌鲁木齐	晴	4	−3
西宁	小雪	5	−4	银川	小雪	0	−3
兰州	雨夹雪	3	−3	西安	小雨	16	7
拉萨	多云	15	1	成都	雷阵雨	17	10
[illegible]	[illegible]	22	11	[illegible]	[illegible]	23	8

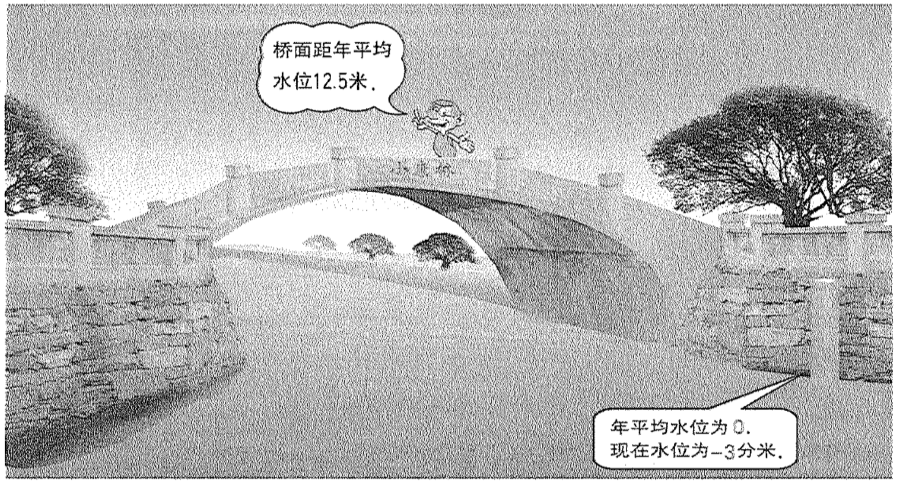
乌鲁木齐的最高温度为 4 ℃，最低温度为 −3℃，这天乌鲁木齐的温差为多少？你是怎么算的？

$$4-(-3)=\ ?$$



6　有理数的加减混合运算

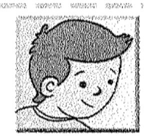
下图是一条河流在枯水期的水位图.



此时小康桥桥面距水面的高度为多少米?

小颖

$12.5-(-0.3)=12.8$(米).



小明

$12.5+0.3=12.8$(米).

你知道小颖和小明分别是怎么想的吗?他们的结果为什么相同?

议一议

一架飞机作特技表演,起飞后的高度变化如下表:

高度变化	记作
上升4.5千米	+4.5千米
下降3.2千米	−3.2千米
上升1.1千米	+1.1千米
下降1.4千米	−1.4千米

此时飞机比起飞点高了多少千米?

$$
\begin{aligned}
&4.5+(-3.2)+1.1+(-1.4)\\
=&1.3+1.1+(-1.4)\\
=&2.4+(-1.4)\\
=&1(\text{千米}).
\end{aligned}
$$

还可以这样计算:

$$
\begin{aligned}
&4.5-3.2+1.1-1.4\\
=&1.3+1.1-1.4\\
=&2.4-1.4\\
=&1(\text{千米}).
\end{aligned}
$$

比较以上两种算法,你发现了什么?

例1 计算:

(1) $-\frac{1}{7}-\left(-\frac{2}{7}\right)$;　　(2) $\left(-\frac{3}{5}\right)+\frac{1}{5}+\left(-\frac{4}{5}\right)$.

2. Introduction to the book (Beijing, 1972)

毛主席语录

我们的教育方针，应该使受教育者在德育、智育、体育几方面都得到发展，成为有社会主义觉悟的有文化的劳动者。

学生也是这样，以学为主，兼学别样，即不但学文，也要学工、学农、学军，也要批判资产阶级。学制要缩短，教育要革命，资产阶级知识分子统治我们学校的现象，再也不能继续下去了。

Introduction to the book (Beijing, 1972)

- Chairman Mao's saying on the aims of education
- The book is written under the guidance of the revolutionary educational thoughts of Chairman Mao
- Learn from the workers, the peasants and the soldiers
- Education needs to be revolutionalized; the past dominion of the bourgeoisie and intelligentsia over education cannot be continued

Hong Kong 1972

INTRODUCTION

Traditionally mathematics has been taught as a collection of isolated and unrelated bits of information with little regard for an understanding of the basic ideas involved. Doubtless, unrelated fragments of knowledge are difficult to retain. This is why mathematics has been hard to learn and hard to remember.

In contrast, mathematics nowadays is no longer a series of isolated facts, but information related to the whole. Mathematics nowaday is viewed as a system in which the individual definitions, assumptions, and properties take their parts as contributing parts of the whole system. Nowadays mathematics must be taught in such a way that students are involved in the development of the mathematical laws, and they are guided so that they can discover the mathematical generalizations themselves. Nowadays students have zeal and joy in learning mathematics, and, because of that, they will retain the mathematical ideas as part of their knowledge.

Accordingly, our main objective in revising our mathematics project are two-fold: understanding of mathematical ideas and the integration of mathematical ideas as a whole.

Our mathematics course from the first form to the upper forms is to be taught solely along two lines:— geometry and algebra. One is built upon the concepts of sets of points, and the other is built upon the concepts of sets of numbers. We are using the same ideas in approaching both. We are repeatedly using the ideas of set, set operations, variables, conditions, one-to-one correspondences, ordered pairs, relations and functions, mathematical systems, name and name-referent and logic.

Among these ideas, the idea of set is the most fundamental and primitive. Unlike other subjects, mathematics is regressive. That is, mathematical ideas and laws are built one upon another in a regressive way, and not in a roundabout way. Thus, in the realm of mathematics we must have a starting point. The starting point involves some undefined terms. The undefined terms are sets, numbers and points. All the other ideas mentioned above are defined in terms of sets, points and numbers. However, it is the idea of sets that enables us to integrate algebra and geometry. What follows will be a short account, relating how we carry out the various ideas mentioned above and how we integrate the different aspects of mathematics with the help of these ideas in the first two years.

Since the idea of set is a primitive idea, set and set operations such as union and intersection are discussed in the first lessons of the first year. In the second year the ideas will be extended to include a study of compound conditions involving 'or' and 'and'. Moreover, the concept of the complement of a set and the logical connective 'not' will be introduced also. These operations will enable the students to find the solution sets of more complicated compound conditions.

Geometry must be studied from two standpoints, namely the non-metric and the metric point of view. At the very beginning of the first year, non-metric geometrical ideas are introduced with the aid of set and set operations. Geometrical figures are defined as sets of points, or intersection of sets, or union of sets. As soon as rational numbers and real numbers are introduced in the second year, geometry will be studied from the metric point of view. Naturally the Pythagorean theorem, other properties of triangles, similar triangles and properties, trigonometric ratios are to be developed along with the measures of plane and space figures both directly and indirectly. In the same year, non-metric geometry will be extended to figures in space.

In algebra, the ideas of set, condition and variable help to unify the presentation of various conditions for equality, and conditions for equivalence. In the first year students learn how to solve problems with conditions in one variable. They will learn how to solve problems with conditions in 2 and 3 variables in the second year with the same ideas of sets, conditions, and variables. The solving of problems of 2 variables will pave the way for the development of the idea of an ordered pair and sets of ordered pairs.

In geometry, a variable is replaced by points from a universe, such as sets of points in a line, a plane, and space. The solution set is a geometrical figure or the empty set.

The idea of ordered pairs and the use of graphs strengthen the integration of the sets of numbers and the set of points. This enables the student to view algebraic conditions from a geometrical point of view, and geometrical conditions from a algebraic point of view. In the first year the idea of one-to-one correspondence is introduced under the topic of natural numbers; and the idea of an ordered pair is used under the topic of rate pairs, proportionality property, and percent. In the second year, the ideas of one-to-one correspondence and ordered pairs will be extended to the ideas of relations and functions. The ideas of relations and functions are to be introduced with the help of graphs. Detailed study of some special graphs will be made in this period. Hence, the graphs of $y = kx$, $y = \frac{k}{x}$,

$y = kx^2$, $y = \frac{k}{x^2}$ are to be studied. The study of relation and function and graphs also leads to the study of formulas in the second year.

Throughout the whole course of mathematics in the first two years, another important concept occurs and reoccurs. This is the idea of the pattern of mathematical systems. A mathematical system consists of a set of elements, some (at least two) operations of these elements, and the properties of these operations. In the first year, the system of natural numbers and rational numbers of arithmetic is introduce In the development of these systems, the students gradually learn that there is a certain pattern from which a number system is constructed. Thus, they not only work with infinite sets of numbers such as natural numbers, but also learn that a finite set of numbers with 2 operations and certain properties of these operations may form a number system. The properties of natural numbers are developed upon the properties of sets and set operations. Upon the properties of natural numbers, the system of rational numbers of arithmetic is developed. The latter is considered to be the extension of the former. This means each system is developed from the previous one and this is the underlying process throughout the whole development of the numbers system. Thus in the second year, the system of rational numbers will be developed as the extension of the system of rational numbers of arithmetic, and the system of real numbers will be developed as the extension of the system of rational numbers.

In revising our programme of mathematics, much attention has been paid to the distinction between names and name-referents. At the very beginning of the first year students are required to make the distinction between numerals and numbers, sentences and statements (or conditions), place-holders and variables. The distinction between names and ideas is an important part of the whole course.

In regard to the method of proof, we should make clear that proof is not carried out vigorously in the first two years. In developing the ideas concerning operations on sets, number systems and properties of operations, logic is used implicitly. This means that proof will be introduced informally. In the second year, deductive proof in both algebra and geometry will be gradually introduced.

In preparing the texts for the Secondary School Course for the English section of St. Paul's Co-Educational College, I have read over a hundred volumes of contemporary mathematics, written by well-known mathematicians. Material have been carefully selected, edited, and revised, so that they could be organized into comprehensive syllabuses for a new project for the whole Secondary School. As references for the readers of my texts, I should like to list out some of the publications which have been the main sources of materials for this project. They are:

Elementary Functions by Halberg Devlin.
Geometry – A Modern Introduction by Keedy Nelson.
Modern Abstract Algebra by R.V. Andree.
Modern Algebra and Trigonometry by E.P. Vance.
Modern Algebra and Trigonometry by Doleiani, Berman, and Wooton.
Seeing Through Mathematics by H.V. Eugen; M.L. Harting etc..
(Book 1, 2, and 3)
Introduction to Modern Algebra by J.K. Kelley.
What is Mathematics by R. Courant and H. Robbins.
Introduction to the Theory of Groups by P.S. Alexandroff.
Elementary Geometry From the Advanced Standpoint by Edwin E. Moise.
Modern Basic Mathematics by H.C. Carter.
Intuitive Concepts in Elementary Topology by B.H. Arnold.
Elementary Contemporary Algebra by Frank Agres.
Matrices by Frank Agres.
Abstract Algebra by J. Fang.
The Mathematics of Matrices by Philip J. Davis.
Linear Algebra by Seymour Lipschutz.
Vector Analysis by M.R. Spoegel.
Geometric Transformations by I.M. Yaglom.
Transformation Geometry by J.J. Friis.
Basic Concepts of Geometry by W. Prenowits and M. Gosdan.
College Calculus by Protter and Morray.
Calculus by Edwinand E. Moise.
Probability by Samuel Goldberg.

In writing the texts I am indebted to many persons. In particular, I should like to express my whole hearted gratitude and thanks to Miss B.M. Kotewall, our principal, to whom I owe my zeal and much encouragement during the preparation of the manuscripts. Then I should like to express my deep appreciation and gratitude to Professor Y.C. Wong, the head of the department of Mathematics of the University of Hong Kong, for his valuable comments and constructive criticism during the development of the project for the 5-year course; to Dr. K.T. Leung from the same department for the advice and suggestions on the detailed syllabuses for forms 3, 4, and 5; to my colleagues Mr. Law Wing-on and Mr.

Kwong Sum Wood for their help; and many other colleagues, who suggested improvements on the arrangement of materials when they were teaching from the manuscripts.

Finally, I should give many thanks to the students who spent many hours typing and re-typing manuscripts, and to those who helped to arrange the manuscripts in order.

Lee Yee.

Hong Kong, September 1968.

Introduction to the book (Hong Kong, 1972)

"Mathematics is viewed as a system in which individual definitions, assumptions, and properties take their parts as contributing parts of the whole system. … Accordingly, our main objectives … are two-fold: understanding of mathematical ideas and the integration of mathematical ideas as a whole. … mathematical ideas and laws are built one upon another in a regressive way … Thus, in the realm of mathematics we must have a starting point. All … ideas … are defined in terms of sets, points and numbers. However, it is the idea of sets that enables us to integrate algebra and geometry."

(then comes a detail description of how the different aspects of mathematics are integrated in this book with the help of these ideas)

Beijing 2005

走进数学新天地

亲爱的同学，祝贺你步入了一个新的学习起点！你将越来越走近数学！

走近数学——

你会觉得生活中处处都有她的身影；

你会发现许多令人惊喜的东西；

你还会感到自己变得越来越聪明，越来越有本领．许多以前不会解决的问题、不会做的事情，现在都能干得很好了；

……

我们将一起走进丰富的图形世界，让你在自己的房间里、教室里、大街上认识许多新的图形，使你在“做一做”“想一想”的活动中发现这些图形的奇妙性质，用它们去设计精美的图案．

我们将一起走进一个全新的“代数”世界，这里有比零小的数，有可以代表任何数的 a，有能够帮你解决许多问题的方法，……学习它们，使用它们，你会感到自己正变得越来越能干．

我们还将学会与身边的数据“对话”，数据会告诉你许多有用的信息，你也能够用数据去表达自己的想法．

先想一想、试一试，再与别人议一议，然后读一读教科书，听一听老师的讲解，这是学好数学的有效方法．

现在让我们一起打开教科书，走进数学新天地吧！

Introduction to the book (Beijing, 2005)

Entering the new world of mathematics

Dear student, congratulations for stepping into a new starting point of learning! You will be moving nearer and nearer mathematics!

You will find mathematics everywhere in life;

You will find many pleasantly surprising things;

You will find yourself smarter and smarter, more and more skillful. Many problems you could not solve in the past, things you could not do, now you can do them very well. ....

Think, try and discuss with others, and then read the textbook, listen to your teacher. This is an effective way of learning mathematics.

Let us open the textbook together, and walk into the new world of mathematics!

3. The first two examples in each book

The first 2 examples in Beijing 1972

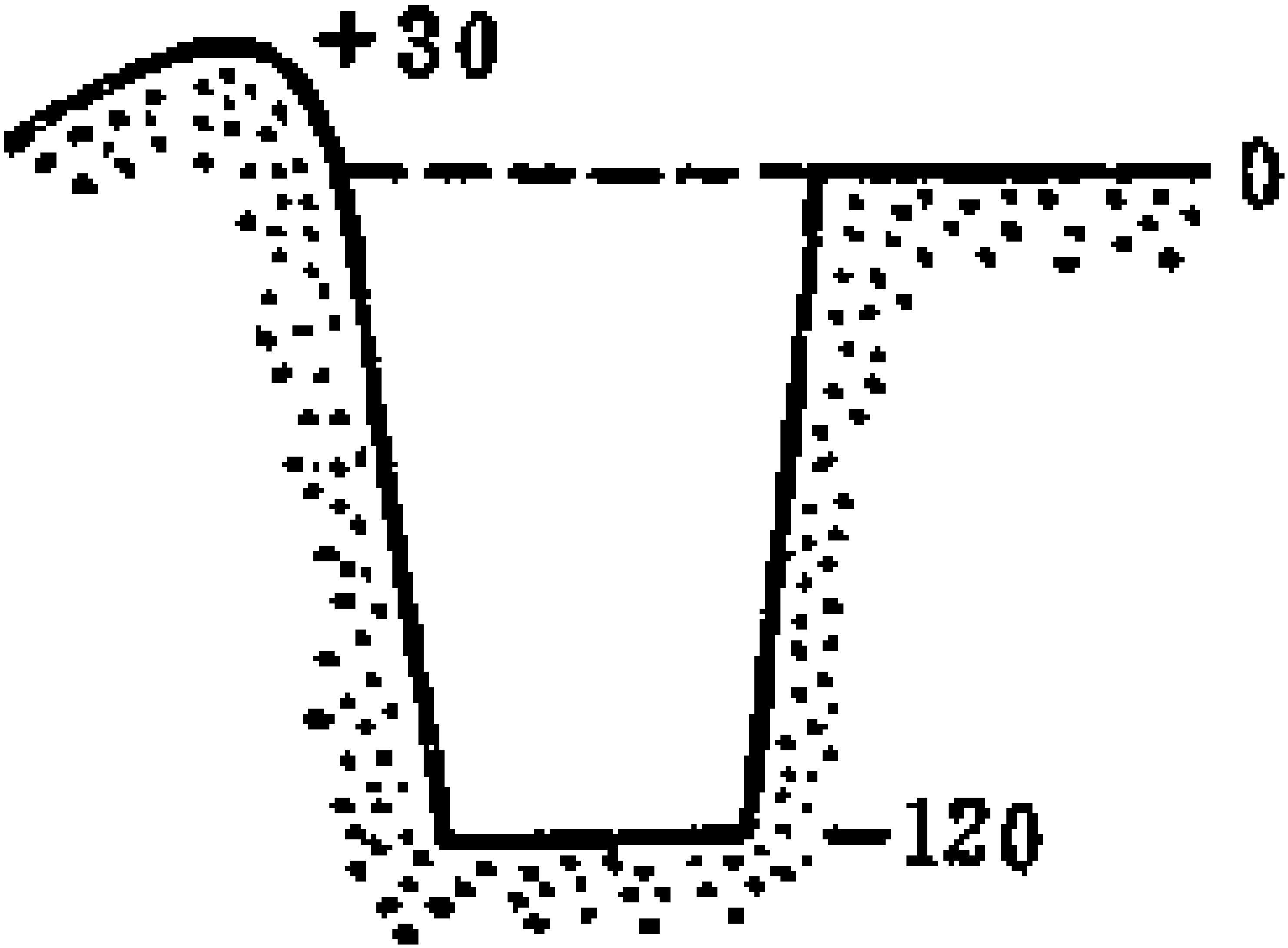
"In the notorious old society, a sheepherder girl is forced to herd the landlord's sheep in the cold, snowy land of 15° C below zero, while the landlord is spending his exploiting and parasitic life in his 15° C house, …"

(Picture)

"A moat needs to be 30 cm above the ground while its base is 120 cm below the ground …"

(picture)


-15°
+15



+30
0
-120

The first 2 examples in Hong Kong 1972

2. NECESSITY OF INTRODUCING NEGATIVE RATIONAL NUMBERS

Consider the following question:

(1) If a and b are natural numbers, is there always a natural number x such that $x + a = b$ will express a true statement?

(2) The temperature of a city at 6 a.m. was 6 1/2°F and at 6 p.m. it dropped by 8 2/3°F . What was the temperature at 6 p.m.?

The first 2 examples in Beijing 2005

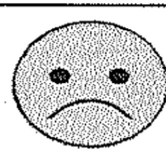
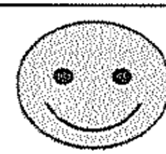
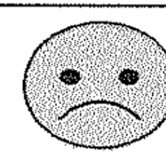
1. A certain class has a quiz, and the scoring guide is as follows: Add 10 marks if one answers an item correctly, deduct 10 marks if one answers an item incorrectly, 0 mark if one does not answer an item. The basic mark for each team (of students) is 0. The way four teams answered the items are as follows:

(picture)

What is the final mark for each team? How do you represent (the marks)? Discuss with your peers.

 加 10 分　 扣 10 分　 得 0 分

某班举行知识竞赛，评分标准是：答对一题加 10 分，答错一题扣 10 分，不回答得 0 分；每个队的基本分均为 0 分．四个代表队答题情况如下表：

	第 1 题	第 2 题	第 3 题	第 4 题	第 5 题
第一队					
第二队					
第三队					
第四队					


最后得分
第一队 第二队 第三队 第四队
10 20 0 10
红色所表示的得分比0低.
最后得分
第一队 第二队 第三队 第四队
10 20 0 －10
带“－”号的得分比0低.

2. In your experience, have you come across numbers with the "-" sign? Discuss with your peers.

Fortune Global 500 world largest retailers				
Rank	Company	Revenues	Profits	Number of employees
2	Wal-Mart Stores	166809.0	5377.0	1140000
46	XXX	46663.6	295.1	171440
66	XXX	39866.7	805.6	297290
111	XXX	30351.9	1088.4	134896
120	XXX	28670.9	423.6	97040
153	XXX	25320.1	-195.2	47953
184	XXX	22451.3	-25.2	34375
Source: "Fortune" Global 500 in year 2000			Unit: million US dollars	

财富全球500强中的主要零售企业

排名	公司	年收入	利润	雇员人数/人
2	沃尔玛	166809.0	5377.0	1140000
46	麦德龙	46663.6	295.1	171440
66	家乐福	39855.7	805.6	297290
111	特斯科	30351.9	1088.4	134896
120	伊藤洋华堂	28670.9	423.6	97040
153	大荣	25320.1	-195.2	47953
184	佳士客	22451.3	-25.2	34375

资料来源:2000年《财富》全球500统计　单位:百万美元

4. Block types and messages

Code	Block Types
N-M	Narrative: Mathematics
N-L	Narrative: Learning and Teaching
N-P	Narrative: Political
<u>Non-Narrative Blocks</u>	
E	Exercise (M, L or P)
W	Worked examples (M, L or P)
G	Graphs or Tables (M, L or P)
Ph	Photos or Pictures (M, L or P)
A	Activities (M, L or P)

Results: Block types and messages

	N-M	N-L	N-P	W	G	Ph	A	E
Beijing 1972	58	10	13	38	24	5	2	435
HK 1972	56	0	0	13	2	0	6	443
Beijing 2005	32	0	1	33	35	35	28	390

Discussion on blocks

- The narrative blocks in all three books are mostly narratives in mathematics (Beijing 2005 has slightly fewer narrative blocks – 33, versus 81 for Beijing 1972 and 56 for Hong Kong 1972)
- All three books contain quite a lot of exercise (Beijing 2005 slightly less)
- Beijing 1972 has more blocks with political messages, and messages about teaching and learning (of mathematics)
- Beijing 2005 has no narrative block on learning and teaching, but has a lot of photos, graphs/tables, and activities (which may convey messages about how mathematics is to be learned)
- Hong Kong: no political blocks, and no photos! (implies that mathematics is apolitical and has nothing to do with the “real” world? – this is in itself a political message!)

5. Types of Exercise

S = Short questions

L = Long questions (numerical)

A = Application problems

T = True or False questions

W = Questions that require explanation, justification or exploration (Why?)

P = Questions of Proof

Number of each type of exercise items

	S	L	A	T	W	P
Beijing 1972	384	1	49	0	1	0
HK 1972	318	8	20	93	2	2
Beijing 2005	268	31	40	9	42	0

Discussion on exercise

- Beijing 1972 has a lot of short numerical items and application items, but very few or no long numerical items, true or false questions, questions that require explanation, justification/exploration, or questions of proof
- Hong Kong 1972 has a lot of true or false questions, but very few questions that require explanation, justification/ exploration, and relatively few application items. It is the only book among the three that has items of proof
- Beijing 2005 has many long numerical items, application items, and questions that require explanation, justification/ exploration

Messages conveyed through the exercise

- Results of the coding are not presented here, but the results are very similar to the findings for the analysis of the narrative blocks
- Some examples of the exercise are given below
- The names of companies etc. used in the exercise may also convey some political messages
  - e.g., "Anti-Imperialism Knitting Company" used in Beijing 1972 (p.59) versus "A certain company" used in Beijing 2005

An application item in Beijing 1972 (p.79)

"Before the Liberation, a capitalist owner of a cotton factory exploited his workers. The workers produced wealth worth \$*a* everyday, but the capitalist only took 5% (of the profit) to pay for the workers' wages. Write down the algebraic expressions for the wages that the capitalist paid the workers per day, and the amount of money that he exploited the workers everyday"

Application items in HK 1972

(only 4 such items (3 to 6 below) in the whole Chapter except Exercise (B) at the very end)

3. What day of the week is Tuesday?
4. What month of the year is September?
5. On which floor of the school building is your classroom?
6. In which row in the class room do you sit?
7. List the names of the rational numbers in the order according to the idea of 'greater than', beginning with the name of the greatest number: $-6\frac{1}{2}$, $-\frac{14}{2}$, $-\frac{15}{2}$, -1, $+\frac{4}{3}$, $+\frac{6}{5}$.

EXERCISE (B)

1. It is found that 270 ft. of shelving are required for every 2000 books. If an increase of 4,500 books are to be added to the library, what is the amount of additional shelving required?

2. Two towns are $3\frac{3}{4}$ cm. apart on a map with a scale of 1 cm. to 1 mile. What is the actual distance between the towns? A road has a length of $11\frac{1}{2}$ miles. What will be its length on the same map?

3. Water weights $62\frac{1}{2}$lb per cubic foot and lead is 11.4 times as dense as water, what will be the weight, in lb, of 1 cubic foot of lead?

4. If it takes me 32 minutes to take a trip from city A to city B when I drive at an average speed of 30 m.p.h., how far apart are A and B?

5. A railway journey between two towns A and B takes $2\frac{3}{5}$ hours with an average speed of 48 m.p.h. If the average speed is increased up to 55 m.p.h., how many minutes will be saved on the journey?

6. In a class of 22 boys and 18 girls, the boys receive an average of four dollars and fifty cents a week in pocket money where as the girls receive an average of five dollars and forty cents a week. Find the average pocket money for the whole class per week.

7. A bridge is $2\frac{2}{3}$ miles long. How long will it take to cross (a) in a car at an average speed of 25 m.p.h.; (b) walking at an average speed of $1\frac{3}{4}$ m.p.h.?

8. After a ball is dropped on a stony floor, it rebounds to $\frac{2}{3}$ of the height from which it is dropped.

(a) How high will it rise at the third rebound if it is dropped from a height of 9.0 ft.?

(b) If it rises to a height of $1\frac{7}{9}$ ft., from what height is it ~~dropped~~?

9. A rectangular yard $12\frac{3}{4}$ ft. long and $6\frac{1}{2}$ ft. wide is surrounded by a road 2 ft. wide. Find the area of the road.

10. The regular price of a refrigerator was \$1500. During a sale, the price was reduced to \$1275. The discount was what per cent of the regular price?

11. Last year Miss. Wang saved \$1860. Her savings amounted to 30% of her salary. What was her salary last year?

12. An atom travels 1.5 meters in 52 seconds. Find its speed in centimeters per second.

13. An iron bar is 10 in. long, 5 in. wide and 3 in. thick. Find its weight if 1 cubic foot of iron weights 450 lb.

14. How many pieces of ribbon can be cut from a coil of ribbon 2 meters long if each piece of ribbon is 6.5 cm. long?

Discussion

- Messages are very explicitly conveyed in the mathematics textbooks during the Cultural Revolution in China
- What kind of mathematical messages is conveyed in these textbooks, in contrast to the textbooks in Hong Kong around the same time, and to textbooks in Beijing today?
- What are the messages about learning mathematics?
- What are the political messages?
- E.g., this is a chapter on directed numbers and rational numbers
- What is a negative number? How is this treated or conceived of differently in the three textbooks?

What are the mathematical messages?

The first example in Beijing 1972

"In the notorious old society, a sheepherder girl is forced to herd the landlord's sheep in the cold, snowy land of 15° C below zero, while the landlord is spending his exploiting and life in his 15° C house, …"

The first example in Hong Kong 1972

2. NECESSITY OF INTRODUCING NEGATIVE RATIONAL NUMBERS

Consider the following question:

(1) If a and b are natural numbers, is there always a natural number x such that $x + a = b$ will express a true statement?

Replace a by 10 and b by 4. Can x be a natural number?

Mathematical messages in Beijing 1972

- Mathematics comes from the real world and applies to the real world
- (Mathematics) concepts are not "a priori", but come from practice
- Engels: "Concepts of numbers and shapes come from the real world and not from anywhere else"
- The applications of mathematics (rather than mathematics being truthful knowledge) are stressed
- Sharp contrast to the view of mathematics in Hong Kong 1972

Mathematical messages in HK1972

- Long trunks of texts with no pictures
- Mathematics is viewed as a study of a system of knowledge, defined in terms of numbers, points and sets, and their operations and properties
- No application is mentioned in the introduction, and there are very few application problems in the exercise
- Many "True or False" questions (and they only appear in this book out of the three) – truthfulness of mathematics is the most important
- Mathematics is absolute truth unrelated to real life?

Message about mathematics learning (Beijing, 1972, p.9)

在社会实践中，就人类认识运动的秩序说来，总是由认识个别的和特殊的事物，逐步地扩大到认识一般的事物。通过温度高低的比较，使我们进一步认识到：

一个数轴上的点越往右边，它表示的数就越大。

As far as human knowledge of the order of events is concerned, (we) always start with individual and specific things, and progressively expand to knowledge of general things. Through comparing temperatures, we know further that the more to the right are points on a number line, the bigger the number.

What are the messages about mathematics learning?

- A dialectic view of learning
- True knowledge comes from practice, through "production struggle", "class struggle" and "scientific experiments"
- It is through "practice" that we learn (mathematics)
- Knowledge is learned from the specific to the general, from the perceptual to the cognitive

What are the political messages?

- The old society is notorious
- In the past, the landlords and Bourgeoisie suppressed the Proletariat (workers, peasants and soldiers)
- Wisdom comes from Chairman Mao
- New society under the leadership of Chairman Mao much more efficient than old society
- Characteristics of the new society: self-reliance, and perseverance
- Instead of true or false in the mathematical sense, there is right or wrong in the political sense

Are these merely jargons of political rhetoric?

- Are the messages in Beijing 1972 merely jargons, to be mentioned because of political necessity?
- Teachers should filter out the jargons and get on to the “pure” mathematics?
- What is “pure” mathematics? Is there “pure” mathematics?
- Of course to learn mathematics, we do not need these political contexts, but is it true that we need a context anyway for all mathematics learning?

Situative learning

- Mathematics has to be learned in context
- Inevitably, we also learn about the context through learning the mathematics
- Many modern textbooks (e.g., Beijing 2005) include a lot of examples on buying and selling – what message does that convey?
- We have already pointed out that HK 1972 conveys a certain philosophical-political message (e.g., Pythagoras Theorem or Gou Gu Ding Li (勾股定理); Pascal Triangle or Yang Hui (杨辉) triangle– what messages do these conveyed?)
- So in this sense, the mathematics in other textbooks is no "purer" than the Beijing 1972 textbooks

The aims of education

- What is (mathematics) education for?
- What is mathematics?
- What is important for our children?
- During the Cultural Revolution in China, this important question was asked: whom shall (mathematics) education (and art (e.g., Peking opera, music)) serve? – serve the people, the Proletariat!
- Whether you agree with the answer to the questions or not, it addresses one important question in mathematics education – why should students learn mathematics?
- We learn mathematics for mathematics' sake (intrinsic motivation), or for other ends?

Consistency between Beijing 1972 & 2005

- This stress on extrinsic (versus intrinsic) motivation in learning mathematics is prevalent in both 1972 and 2005
- And both stress the applications of mathematics
- In both cases, we are to learn mathematics in order to serve the country and the people
- In 1972, we serve through "revolution", in 2005, we serve through building up the economy
- (Other common values: uniformity, orthodoxy)
- Consistent with traditional Chinese values - Embodiment of message in the text (文以載道)
- Even the Cultural Revolution is unable to totally revolutionize the deep-rooted Chinese cultural values
- It is still old wine in new bottles

Conclusion

- Even mathematics textbooks not produced at a time of political upheaval are still conveying the values of the dominant authorities, albeit in a more implicit manner.
- This testifies to the intricate relation between the mathematics textbook and the underlying values of the education systems concerned, and corroborates the assertion that the mathematics textbooks can be regarded as a cultural artifact
- It also echoes the fact that all aspects of education, including textbooks, are under the strong (but maybe subtle) influence of the underlying culture

Implications

- For both curriculum developers and classroom teachers, we have to realize that textbooks are socio-cultural and political products
- Should examine the values conveyed in textbooks, not just the mathematics
- We may have our own values and “messages”, but we should make them explicit o students, and allow them choice and encourage them to choose
- We should also help students to be aware of these values, and develop their own values

Concluding remarks

- The textbooks in the Cultural Revolution period are in themselves interesting
- But more important is the fact that what happened in this extreme period of time in the history of mankind brings out starkly the fact that the textbook is but a cultural product
- It conveys, intentionally or unintentionally, messages other than mathematics
- We mathematics teachers should not naively believe that we are "purely" mathematics teachers
- Like it or not, we are teaching our students more than just mathematics
- If we are more conscious of this fact, we may equip ourselves not only to be better at teaching mathematics, but also better at teaching through mathematics

Comrades: I thank you very much for your attention!

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