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FROM CLAY TABLET TO COMPUTER TABLET: THE EVOLUTION OF SCHOOL MATHEMATICS TEXTBOOKS

Jeremy Kilpatrick

University of Georgia

jkilpat@uga.edu

Over the centuries and around the world, school mathematics textbooks have differed in many ways. In this idiosyncratic survey, I attempt to portray, across time and space, something of what researchers have learned about those textbooks: what they are, what they appear to be, how they are related, and how they have been used. In general, school mathematics textbooks have differed more in approach and form than in function or content. Their principal function has been to serve as repositories of authorized knowledge, although at times they have been enlisted as resources for creative problem solving or as material for self-instruction. In the past, as textbooks took different forms and appeared in different media—clay tablet, papyrus, parchment, bamboo, paper—they also began to take on a wider span of mathematical content and to vary extensively in pedagogical approach. Research on textbooks has examined many of their characteristics, looking at how they have changed over time and, less often, how they differ across communities. Today, school mathematics textbooks seem more similar in mathematical content than they are in appearance, pedagogical outlook, or assistance for the teacher. There does seem to be something of a virtual school mathematics curriculum worldwide, whereas there is little agreement on what features the textbooks enshrining that curriculum should have. Today's textbooks vary along many dimensions. It appears that textbooks are being written as much or more for the teacher as they are for the learner. Tomorrow, computing technology seems likely not only to yield tailor-made textbooks but also to allow authors and publishers to revise the text swiftly in light of how the learner responds.
Key words: school mathematics textbooks, form, function, content, pedagogy

INTRODUCTION

Just about every author who attempts to survey the research on school textbooks begins by noting the somewhat surprising scarcity of such research. That scarcity seems due, in part, to uncertainty as how to define *textbook* but also to its somewhat questionable status in school instruction. "Today a number of countries have curricula for primary and lower-secondary schools which literally never mention the word textbook" (Johnsen, 1993, p. 22). As long as educators remain unsure as to whether textbooks actually benefit school instruction, "we hesitate to apply terms like science and research to a field whose existence has not yet been sanctioned" (p. 22). Nonetheless, there appears to be a growing multidisciplinary literature on the nature and content of textbooks as well as how they are seen and used.

Egil Johnsen (1993) categorized international research on textbooks using three categories:

1. Ideology in textbooks.
2. The use of textbooks.

3. The development of textbooks. (p. 28)

These categories—which can be seen as oriented toward *product*, *use*, and *process*, respectively—are listed in the order in which they have been the focus of research to date, from most to least. In the case of mathematics textbooks, content analyses in the first category have typically given somewhat more attention to the selection and presentation of mathematical content than to philosophical underpinnings of that selection and presentation. But certainly the preponderance of research on mathematics textbooks has examined the contents of textbooks as a product. Research into the ways in which mathematics textbooks are used by teachers and learners has been growing in recent decades, with increased attention to the effectiveness of various uses. In contrast, the process of developing mathematics textbooks—from their conceptualization, writing, and editing to their marketing, selection, and distribution—has received relatively little attention to date.

WHAT IS A TEXTBOOK?

The usual definition of *textbook* is something like the following: a book used for the study of a subject. With that definition, a *mathematics textbook* becomes a book used for the study of mathematics. People who do research on textbooks, however, often observe that coming up with a good definition is not so simple, and they sometimes end up using terms like *teaching media* or *text materials* instead because they do not want to be restricted to a bound collection of printed sheets of paper produced for use in instruction. For example, a 1955 study from the University of Illinois acknowledged that "in recent years . . . text*books* have been supplemented by a variety of other text materials" (Cronbach, 1955, p. 17). Recognizing that devices such as films and recordings could be seen as functioning like texts, the committee conducting the study "restricted its interest to printed text materials of the sort which can be placed in the hands of every pupil" (p. 17).

In recent years, the meaning of *textbook* has expanded to include material produced in an electronic format. For example, in 2011, the state of Indiana defined *textbook* as follows:

> "Textbook" means systematically organized material designed to provide a specific level of instruction in a subject matter category, including: (1) books; (2) hardware that will be consumed, accessed, or used by a single student during a semester or school year; (3) computer software; and (4) digital content.
> (see http://www.in.gov/legislative/bills/2011/HE/HE1429.1.html)

Even before the electronic age, however, textbooks were not necessarily bound collections of pages. Over the centuries, textbooks have appeared in different media—clay tablet, papyrus, parchment, and bamboo—as well as paper. The word *textbook* (a blending of *text* and *book*) apparently did not appear until about 1830 (Stray, 1994, p. 1), with *text* being a word from the 14th century that had the meaning "thing woven" (from the Online Etymology Dictionary at http://www.etymonline.com). Ancient civilizations, however, certainly had equivalents of our modern books. Clay tablets with mathematical content from ancient Mesopotamia were used as school texts (Høyrup, 2002, p. 8), as was the *Nine Chapters on the Mathematical Art* (Shen, Crossley, & Lun, 1999), which "served as a textbook not only in China but also in neighbouring countries and regions until western science was introduced into the Far East around 1600 AD” (p. 1).

Gutenberg’s invention of the modern printing press in the 1450s enabled textbooks to be printed more cheaply than before, but it took many years before they were available to all pupils. Instead, as Nerida Ellerton and Ken Clements (2012) have noted, beginning in Western European nations in the 16th century and moving to North America in the 17th century, pupils prepared and used so-called cyphering books, linking writing and penmanship to the learning of mathematics. It is likely that in North America even by the 18th century

> most students outside of the large cities did not own commercially-printed arithmetic books (Meriwether, 1907; Small, 1914). Some teachers might have owned their own commercially-printed arithmetic texts, but most of them based their methods of instruction on old cyphering books that they had copied or procured from other masters (Littlefield, 1904). Indeed, William Munsell (1882) claimed that well into the 19th century printed arithmetics were so scarce that many scholars never saw one. That appears also to have been true in relation to the use of printed arithmetics in schools in Great Britain in the early 19th century (Williamson, 1928). (Ellerton & Clements, 2012, p. 107)

During the 19th and 20th centuries, bound textbooks printed on paper became increasingly common fixtures of school instruction world-wide. As they became more widespread, such books began to be produced and used in a greater variety of ways. Stray (1994) characterizes the textbook as “a composite cultural commodity which provides an authoritative pedagogic version of received knowledge” (p. 4). That characterization captures some of the ways in which textbooks operate as “the intersection of several relationships involving teachers and pupils, producers and consumers, institutions and the state” (p. 24). In the evolution of mathematics textbooks, there have been changes on the inside (what the textbooks contain) and on the outside (how they are produced and used)—more changes than can be captured in a paper such as this. What follows is a brief sketch of some of the differences those changes have produced.

DIFFERENCES IN CONTENT AND FUNCTION

School mathematics textbooks differ in content, function, approach, and form. In general, they seem to be more similar in the first two dimensions than in the second two. They have certainly grown in the contents they contain and the functions they serve, but they still share many common features in those respects.

Contents of Mathematics Textbooks

During the so-called new math era—roughly the mid1950s to the mid1970s—one assumption that reformers made was that

> the curriculum would be brought up to date mathematically if they could simply get their new syllabuses and textbooks into the hands of students and teachers. By the end of the era, they had come to see that much more was required. At the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it with his or her pupils—all situated in a specific educational and cultural context.

> From a distance, school mathematics looks much the same everywhere. Countries include many of the same topics in their syllabuses and expect pupils to solve many of the same sorts of problems. International comparative studies are predicated on having a common framework on which to base the assessment of mathematics achievement. Up close, however, each country has a unique school mathematics. (Kilpatrick, 2012, p. 569)

Research studies conducted during the new math era found that changes made in textbook content did not necessarily translate into changed performance by learners. For example, the results of the National Longitudinal Study of Mathematical Abilities, a 5-year study conducted in the United States that began in 1962,

> showed that, contrary to some expectations, pupils who had used modern textbooks did not achieve superior levels of performance in computation over several years while using textbooks that did not stress computation. . . . To oversimplify, pupils tended to learn what was emphasized in the textbooks they used and not something else. (Howson, Keitel, & Kilpatrick, 1981, p. 193).

International studies such as TIMSS [Trends in International Mathematics and Science Study] depend on mathematics textbooks being roughly equivalent over time and across countries. Analyses such as that by Vilma Mesa (2009) call that equivalence into question when one looks closely at textbook content. But international studies do not look closely at content. They depend on the existence of something akin to an

> idealized international curriculum. No allowance is made for different aims, issue, history and context across the mathematics curricula of the systems being studied. (Keitel & Kilpatrick, 1999, p. 243)

There does seem to be something of a virtual school mathematics curriculum worldwide, whereas there is little agreement on what features the textbooks enshrining that curriculum should have.

Functions of Mathematics Textbooks

Like every curriculum, every textbook represents a selection from available knowledge. "In the process by which a message gets from the resources of civilization to the student" (Cronbach, 1955, p. 93), a sequence of gatekeepers passes on some messages and blocks others. That sequence can be represented as follows: resources→(writer)→manuscript→(publisher)→printed materials→(distribution & selection)→book in school→(teacher)→messages before learner→(learner)→messages noted and stored (p. 94). Each item in parentheses is a gatekeeper that selects the messages to be passed on. The messages that arrive at the learners, therefore, are a function not only of the text material but also of the teacher, and the function those messages serve in instruction depends very much on how the teacher views the text.

Throughout history, the principal function of mathematics textbooks has been to serve as repositories of authorized knowledge. The content messages they contain have been chosen by their authors to represent the most reliable and important mathematical knowledge that the culture possesses. At times, however, authors have attempted to move away from being formal and comprehensive to enable learners to play a part in developing mathematical ideas on their own. Textbooks have been enlisted as resources for creative problem solving or as material for self-instruction. Those functions can rock the pedestal on which textbooks are placed as authoritative. For example, Hung-Hsi Wu (1997) faulted a precalculus textbook for omitting the formula relating degrees and radians, leaving it as a problem for students to solve.

> The issue here, as any textbook author will recognize, is the tension between textbook as archive and textbook as tool for learning. Once a formula is put into a text for memorization and subsequent reference, there is little point in asking the reader to find it. (Kilpatrick, 1997, pp. 958–959)

Textbooks are often seen as dictating the content of instruction, but that appears to be questionable. In a study of four fourth-grade teachers who kept daily logs of their mathematics lessons for a year, Donald Freeman and Andy Porter (1989) found little overlap between the topics the teachers said they taught and the topics in the textbooks they used. Only one of the teachers followed the textbook closely, and she did not teach every topic in the book, skipping or not reaching almost 40% of the lessons. Further, the time that the teachers spent on topics did not match the emphasis those topics were given in the textbooks. This issue seems to be one deserving more attention by researchers.

Mathematics textbooks typically reflect national or other values, and one of their important functions is to attempt to instil those values (Howson et al., 1981). In the first half of the 19th century, for example, Dutch mathematics textbooks began to include dialogues between teacher and learner, giving advice on the best way to study and holding out eagerness and perseverance as important middle-class values for students to acquire (Beckers, 2000). The problems given to learners to solve were not necessarily realistic, but they were designed to incorporate knowledge from fields such as history and geography. Narratives were given in which good behavior such as saving money was rewarded and bad behavior punished. Learners were to become good citizens not only by learning to reason but also by learning right from wrong. It seems that textbooks invariably attempt to transmit their authors' and publishers' beliefs as received knowledge.

DIFFERENCES IN FORM AND APPROACH

School mathematics textbooks seem more similar in mathematical content than they are in appearance, pedagogical outlook, or assistance for the teacher. They appear in a variety of forms and adopt a variety of approaches to the presentation of content.

Forms of Mathematics Textbooks

The earliest textbooks in the Babylonian scribal schools were collections of arithmetic tables or of problems, most of which involved finding a number that satisfied given conditions (Høyrup, 2002; see also Aaboe, 1964; Beery & Swetz, 2012). The problem collections varied in approach: Sometimes the solution was given, and sometimes the text outlined the procedure to be followed in reaching that solution. They were clearly intended to be used in schools, and presumably a teacher was expected to explain the procedure and set the assignment. Today's textbooks vary in a similar fashion but along many more dimensions.

The physical form that a textbook takes has often been seen as part of its value for instruction. In an early comparative study of lower secondary mathematics textbooks, a group of teachers in the Los Angeles, CA, public schools evaluated the books on a variety of criteria. These included not simply the mathematical content but also the number and kind of illustrations; drills, tests, summaries, and reviews; vocabulary; size of numbers; physical features; authors; and matters of organization and presentation (Fuller, 1928). The goal was to determine which textbook ought to be adopted by the district. Although many of the criteria—especially those dealing with the physical form of the books—were easy to apply, it is difficult to see how they should be synthesized to arrive at the optimal choice of textbook.

In a recent related but more sophisticated effort, the Nordic countries—in which textbooks are heavily used—have established a network of researchers to look at issues such as how textbooks are used, hidden messages in the text, and how language and pictures are used (Grevholm, 2011). The network promises to fill a long-standing gap in studies of how the form of a textbook might influence its use and how learners respond to it. A study by Karen Fuson and Yeping Li (2009) provides one example of how the physical forms of textbooks might be analyzed and compared. Fuson and Li examined features of two textbook series—one in China and one in the United States—and found striking differences between the series in the linguistic, visual, and numerical supports provided in the books. The forms that mathematics textbooks are taking should continue to be an active topic of research.

Approaches of Mathematics Textbooks

Textbooks continue to vary extensively in pedagogical approach: from providing no suggestions at all to including detailed scripts specifying what the teacher should say and do. Research on textbooks has examined many of their characteristic approaches, looking at how they have changed over time and, less often, how they differ across communities.

One of the first, and still very few, studies of changed approaches to the content of mathematics textbooks was conducted by Hobart Heller (1940), who surveyed the topic of factoring in algebra textbooks published in England and the United States from 1631 to 1890. Why did factoring grow in emphasis during that time? Heller proposes that it grew in part because examinations began to include more complex problems dealing with forms used in fractions. “Increased complexity of fraction exercises demanded increased emphasis upon factoring. . . . In brief, the materials of factoring and fractions were in a ‘vicious circle’” (p. 110). Factoring also became more prominent among the exercises students were given to work because textbook authors, failing to distinguish between the functions of “illustrative examples” and “model examples,” began to put increased emphasis on drill exercises set to follow a model in place of broader aims for teaching algebra. Heller’s study shows how tradition appears to govern the persistence of the approach to content in textbooks.

Peter Damerow (1980), who looked at geometry textbooks in Germany during the decade of curriculum reform preceding 1980, made a similar finding. Although the new math reforms were intended to offer a new way of thinking about mathematics, simplifying topics by introducing structural ideas, Damerow’s study showed that the quantity of information grew without being reorganized so that it might be approached differently. “As far as Germany is concerned, [the findings] demonstrate that the changes in the textbooks to a great extent do not correspond to the ideas of the reform movement" (p. 281). After examining the nature and number of theorems related to the topic of similarity in the available textbooks, Damerow found the following:

> Instead of exchanging the old conceptual structures against the new ones there were simply added theorems formulated with new concepts. Moreover the added conceptual structures correspond rather to the ideas of Felix Klein introduced at the beginning of the 20th century than to the ideas of Bourbaki as they are defining features of the intended new math curriculum. (p. 301)

In addition to studies of changes in the approach to mathematical topics over time, there have been some studies of differences across communities—in particular, national differences in the way in which certain topics are treated. For example, Tony Harries and Ros Sutherland (2000) examined how primary mathematics textbooks represent multiplication in England, France, Hungary, Singapore, and the United States. (The last country is not discussed in the paper.) They found that the Hungarian, French, and Singaporean textbooks tried, in different ways, to support the development of mathematical meaning by moving from a real situation to a mathematical representation, whereas the English textbooks did not follow a coherent progression.

Mesa (2009) looked at the treatment of the concept of function in textbooks from 18 countries. She found that within and across the countries, there were four clusters of textbooks: rule oriented, abstract oriented, abstract oriented with applications, and applications oriented. These clusters were not aligned with cultural similarities across countries, nor were they necessarily characteristic of individual countries. She concluded:

> When it comes to functions, there may be no such thing as a canonical curriculum in school mathematics. It seems to be false—and this result is also supported by the TIMSS curriculum analysis—that mathematical content is expressed in the same way across the globe. (p. 116)

The approaches to instruction taken in mathematics textbooks continue to pose open questions. Constance Dooley (1960) analyzed the content and approach of textbooks and concluded that the market for U.S. arithmetic textbooks is reasonably sensitive to the results of recommendations from research. That raises the question of how research results are being used, or not used, today.

Another question is for whom are textbooks written? Johnsen (1993) claims that "in principle, today's textbooks are not written for teachers" (p. 323), and he cites a survey of Norwegian textbook authors responding to a questionnaire who said that they write for the pupils. Peggy Kidwell, Amy Ackerberg-Hastings, and Dave Roberts (2008), in contrast, trace the transformation of American textbooks "into a common tool and lucrative product" (p. 4), suggesting that over the years, the marketplace has done much to determine the nature of school mathematics textbooks. It appears that, despite what textbook authors might claim, textbooks continue to be written as much or more for the teacher as they are for the learner.

TOMORROW'S TEXTBOOKS

Koeno Gravemeijer (in press) discusses the need to transform mathematics education in the 21st century but simultaneously notes the limitations of current textbooks as means of making that transformation. He says,

> The scripted character and the ready-made tasks of conventional textbooks limit teachers in adapting to their students' reasoning. Instead, textbooks will have to inform teachers about local instruction theories, and explicate what mental activities hypothetical learning trajectories have to focus on.

He also argues that textbooks will need to be more explicit about the theories being used and will need to contain exemplary instructional activities. He notes that such textbooks may be accompanied by computers and other tools. New textbooks will be far from enough, however, if school mathematics is to be transformed. Teacher education and professional development should accompany any changes in text materials. Gravemeijer claims that "textbook design and [teacher] professionalization will have to be carefully aligned to be successful."

Tomorrow, computing technology seems likely not only to yield tailor-made textbooks but also to allow authors and publishers to revise the text swiftly in light of how the learner responds. That change promises to open up new vistas in the contents, functions, forms, and approaches of mathematics textbooks. In particular, interactive computer textbooks will allow learners—just like those Babylonian learners using clay tablets several millennia ago and those Western European and North American learners using cyphering books several centuries ago—to create their own text. Tomorrow's learners will be able to get instant feedback on their mathematical work from a human teacher or a sophisticated computer program, and instruction can then be tailored to their abilities and interests.

We cannot, however, count on the change to be as rapid as we might like:

> It's typical for new technologies initially to mimic an existing one: Gutenberg's forty-two-line Bible is not easy to distinguish from a manuscript copy. It takes time to figure out what a new medium can do besides the same thing bigger, faster, or cheaper, and for its particular strengths and weaknesses to emerge. Fifty years after Gutenberg, printing had shown itself vastly superior for Bibles and legal texts, a cheap substitute for deluxe books of hours, and no replacement at all for wills, inventories, and personal letters. (Hays, 2014, p. 20)

The strengths and weaknesses of the computer tablet as mathematics textbook remain to be discovered.

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