

# **A Comparative Analysis of National Curricula Relating to Fractions in England and in Taiwan**

Hui-Chuan Li

University of Cambridge

Yan-Shing Chang

Kings' College, London

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# Three Parts of My Presentation

## **1. Introduction**

- Why look at fractions?
- The aims of this study

## **2. Method**

- What curricula are included?
- The four areas of fractions

## **3. Results and Implications**

# **Introduction**

# Why look at fractions?

The teaching and learning of fractions is not only very hard; it is, in the broader scheme of things, a dismal failure (Davis et al., 1993, p.63).

Such a quote seems shocking, but, unfortunately, other research verifies their statement (e.g. Li, in press; Moseley, 2005).

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Davis, G., Hunting, R. P., & Pearn, C. (1993). What might a fraction mean to a child and how would a teacher know?. *Journal of Mathematics Behavior*, 12(1), 63-76.

Li, H.-C. (in press). A comparative analysis of British and Taiwanese students' conceptual and procedural knowledge of fraction addition. *International Journal of Mathematical Education in Science and Technology*. doi: 10.1080/0020739X.2014.892163

Moseley, B. (2005). Students' early mathematical representation knowledge: the effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics*, 60(1), 37-69.

# Why fractions are so difficult for many students?

## 1. Students learn algorithms about fractions without meaningful grounding in this area

For many students, they may be able to use algorithms to compute with fractions, but they do not necessarily understand the concepts that underlie the operations (e.g. Li, in press; Reys & Yang, 1998).

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Li, H.-C. (in press). A comparative analysis of British and Taiwanese students' conceptual and procedural knowledge of fraction addition. *International Journal of Mathematical Education in Science and Technology*. doi: 10.1080/0020739X.2014.892163

Reys, R. E., & Yang, D.-C. (1998). Relationship between computational performance and number sense among sixth- and eighth-Grade students in Taiwan. *Journal for Research in Mathematics Education*, 29(2), 225-237.

# Why fractions are so difficult for many students?

## 2. Students have a limited view of fractions

Students' concept of fractions tends to be mainly dominated by the part-whole construct, and tends to neglect other constructs of fractions (e.g. Chan et al., 2007; Izsák et al., 2008).

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Chan, W.-H., Leu, Y.-C., & Chen, C.-M. (2007). Exploring group-wise conceptual deficiencies of fractions for fifth and sixth graders in Taiwan. *The Journal of Experimental Education*, 76(1), 26-57. doi: 10.3200/JEXE.76.1.26-58

Izsák, A., Tillema, E., & Tunç-Pekkan, Z. (2008). Teaching and learning fraction addition on number lines. *Journal for Research in Mathematics Education*, 39(1), 33-62. doi: 10.2307/30034887

For example, the fraction:

$$\frac{3}{4}$$

can represent three parts of four parts but also  
can mean

- A ratio of two natural numbers
- A number in its own right
- A representation of division
- An operator ( $\frac{3}{4}$  of something is different from the number  $\frac{3}{4}$ )
- Equivalent to  $\frac{6}{8}, \frac{9}{12} \dots$
- $\frac{3}{4} = 0.75$  ,  $\frac{3}{4} = 75\%$

## **British and Taiwanese pupils' knowledge of fractions**

In my previous work (Li, in press) of 561 British and 648 Taiwanese pupils' knowledge of fractions at ages 12 and 13:

- In terms of procedural skills: the Taiwanese students are able to apply algorithms to do fraction calculations far more successfully than their British peers.

## **British and Taiwanese pupils' knowledge of fractions**

- In terms of conceptual understanding:
  - While the Taiwanese students also performed better than their British peers,
  - it was also found that in both group, students' conceptual understanding of fractions is confused and incomplete, even those who are successfully able to perform the operations.

# Importance of maths curriculum and textbooks

- The mathematics curriculum and textbooks play an important role in pupils' opportunity to learn mathematics (Fan & Zhu, 2007; Lui & Leung, 2013).
- Studies of curriculum and textbooks can benefit from comparative, cross-cultural research since many issues can be revealed more clearly (Kulm & Li, 2009).

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Fan, L., & Zhu, Y. (2007). From convergence to divergence: the development of mathematical problem solving in research, curriculum, and classroom practice in Singapore. *ZDM-International Journal on Mathematics Education*, 39, 491-501.

Kulm, G., & Li, Y. (2009). Curriculum research to improve teaching and learning: national and cross-national studies. *ZDM- International Journal on Mathematics Education*, 41, 709-715.

Lui, K. W., & Leung, F. K. S. (2013). Curriculum traditions in Berlin and Hong Kong: a comparative case study of the implemented mathematics curriculum. *ZDM -International Journal on Mathematics Education*, 45, 35-46.

# The aims of this study

This study seeks

- To provide insights into the similarities and differences in the national curricula between England and Taiwan.
- To cast light on how the intended curricula vary and what can be learned from this comparison, in particular for mathematics textbook development and research.

# **Method**

# What curricula are included?

In Taiwan:

- The “Grade 1-9 curriculum” in Taiwan that was published in 2003 (hereafter, called Taiwan (2003)).
- Implemented from 2005 to the present.
- For pupils aged 6-15

# What curricula are included?

In England:

- The current national curricula: the 1999 and 2007 National Curriculum in England that were published in 1999 and 2007, respectively (hereafter, called England (1999, 2007)).
- For pupils aged 5-16

# What curricula are included?

In England:

The new 2013 National Curriculum in England that was published in 2013 (hereafter, called England (2013)).

- It will be implemented from September 2014.
- At the moment, it was only published for pupils aged 5-14.

# The four areas of fractions

The content that relates to fractions is divided into four areas:

- The concept of fractions: the part-whole, quotient, measure, ratio and operator construct (Kieren, 1988).
- Equivalence between fractions to compare and order fractions.
- The relationship between fractions, decimals and percentages.
- The four operations with fractions.

# Results

# Results

Similarities:

All of these curricula

- cover the similar breadth of content and expectations in children's learning of fractions.

# Results

## Differences:

- The differences between the curricula are mainly found in the way the content is broken down and sequenced.

# Sequence of fractions across curricula

Table 1: Sequence of fractions across curricula for pupils aged between 5 and 14

	Taiwan (2003)	England (1999, 2007)	England (2013)
The concept of fractions	Ages 7-11	Ages 7-11	Ages 5-11
Equivalence between fractions to compare and order fractions	Ages 8-10	Ages 7-10	Ages 6-10
Relationship between fractions, decimals and percentages	Ages 10-11	Ages 9-14	Ages 9-14
The four operations with fractions	Ages 8-12	Ages 11-14	Ages 7-14

## Summary of findings

In Taiwan (2003) curriculum:

The concepts and skills of fractions are integrated for primary pupils at ages 7-12.

- Because it intends to equip their pupils with necessary fraction knowledge before they enter junior high school at age 12 (MOE in Taiwan, 2003).

## Summary of findings

In England (1999, 2007) curriculum:

The concepts and skills of fractions are divided into two steps in England (1999, 2007).

- The concepts of fractions are introduced from primary schools.
- The skills of fractions are introduced from secondary schools.

## Summary of findings

It is clear that:

- The introduction of calculations with fractions varies significantly between Taiwan (2003) and England (1999, 2007).
- This may partially explain Li's (in press) finding that at ages 12-13, Taiwanese pupils show greater fluency in operations with fractions than their British peers.

## Summary of findings

It is also clear that

England (2013) makes a significant change in the sequence regarding fraction calculations.

- Pupils in England are expected to add and subtract fractions with the same denominator (within one whole) from age 7.

- The new 2013 curriculum introduces fractions to pupils from much earlier among the three curricula.

	Taiwan (2003)	England (1999, 2007)	England (2013)
The concept of fractions	Ages 7-11	Ages 7-11	Ages 5-11
Equivalence between fractions to compare and order fractions	Ages 8-10	Ages 7-10	Ages 6-10
Relationship between fractions, ratios, decimals and percentages	Ages 10-11	Ages 9-14	Ages 9-14
The four operations with fractions	Ages 8-12	Ages 11-14	Ages 7-14

## Summary of findings

In general,

Taiwan:

- An example of an “intensive” curriculum in the area of fractions.
- Both conceptual and procedural knowledge of fractions are introduced to pupils by the end of their primary years.

# Summary of findings

England:

- An example of an “extensive” curriculum in the area of fractions.
- Fractions are introduced to pupils over the course of primary and secondary schooling.

# **Implications**

# Implications

1. Mathematics textbook development relating to fractions following England's new 2013 curriculum may consider:
  - How best to introduce fractions concepts and operations to pupils as young as ages 5 and 7 respectively.
  - How best to support teachers' teaching strategies since they may not be equipped to teach fractions to pupils at a younger age.

# Implications

2. Researchers interested in textbook comparison and pupil attainments may examine the impact of topic sequence on children's learning of fractions across textbooks:
  - To what extent, an “intensive” or “extensive” content of textbooks on fractions in England and Taiwan helps pupils develop their understanding of fractions.

# Implications

3. There has been a lack of research focusing on the process of textbook development (Fan et al., 2013).
  - The changes in England's new curriculum in fractions may provide a great opportunity for textbook researchers to centre on how fractions-related content of textbooks is produced.

**Thank you very much**