

Technological resources that come with mathematics textbooks: potentials and constraints

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Summary

- Introduction
- Theoretical framework
- Methodology
- Learning environment
- Implementation of the learning environment
- Data analysis
- Final remarks

Introduction

- Traditionally in educational systems:
 - there is a strong tradition of using textbooks;
 - occasionally textbooks are accompanied by technological tools (CD-ROMs, web pages or learning platforms);
 - the curriculum modelled by teachers is based mainly on the use of the textbook adopted in paper format;
 - often teachers use some technological tools associated with textbooks, especially those that do not involve student's manipulation (eg. ppoints);

Introduction

- This presentation is about the use of technological tools in the context of the classroom.
- It is based on the curriculum presented to teachers, through textbooks and electronic materials and the aim is to explain how these materials become learning tools.
- This study is based in one of the technological mediators available by one of the Portuguese publishing houses, *Escola Virtual*, that is highly structured, following the sequence of learning presented in the textbook on paper.

Theoretical framework

- *Activity Theory* (Vygotsky, Leont'ev), assuming its system of collective activity (object oriented and mediated by artifacts). (Engeström, 2001)
- The concept of *instrumental genesis* that involves two processes: *instrumentalization* and *instrumentation*. These processes enable the development and evolution of the instruments. (Rabardel, 1995)
- The notion of *documentational genesis*. It is a construct that expands the concept of artifact by defining the notion of document as building utilization schemes in teachers action mediated by didactical resources. (Gueudet and Trouche, 2012)

Methodology

- This study follows a methodology of qualitative nature and is based on two case studies.
- Involve two groups of secondary school students that use mainly textbooks, and this was the first time that they used the electronic resource as a learning tool.
- Each case study involves several work sessions with the tool. One case is centered on the theme of functions and the other in geometry.

Methodology

- Students are organized in groups of two.
- Working sessions were recorded by special software that records sound and all actions carried out on the computer screen.
- Contents have not been addressed previously by the teacher.
- The process of instrumental genesis started simultaneously with the learning process.

Learning environment (*Escola Virtual*)

- Two topics:
 - quadratic functions;
 - Cartesian geometry.
- These case studies are based on the use of a CD ROM

Objetivos

Objetivos:

- 1 - Identificar uma função quadrática.
- 2 - Representar graficamente uma função quadrática de modo a observar as suas principais características.
- 3 - Identificar o sentido da concavidade da função quadrática por observação da sua expressão designatória.
- 4 - Identificar as propriedades da função quadrática.
- 5 - Identificar o vértice e o eixo de simetria da parábola que representa graficamente a função quadrática.

- Introdução
- Propriedades da função quadrática
- Concavidade do gráfico de uma função quadrática
- Comportamento do gráfico das funções quadráticas
- Determinação das coordenadas do vértice da parábola
- Determinação das coordenadas do vértice da parábola
- Domínio, contradomínio, intervalos de monotonia e sinal
- Exercícios

Objetivos

Objetivos:



- Introdução
- Referencial Cartesiano - Ortogonal e Monométrico
- Simetrias no plano
- Retas paralelas aos eixos
- Semiplanos
- Condições no plano: negação, conjunção e disjunção de condições
- Primeiras Leis de De Morgan
- Exercícios I
- Exercícios II

Implementation of the learning environment

- Initially students followed the structure of the lesson presented in the tool, with some differences:
 - students who studied the quadratic function had a guiding worksheet to explore the tool;
 - students who have studied geometry followed the topics presented in the tool and then they used a worksheet with exercises provided by the teacher.
- Students have appropriated quickly the use of the tool and the instrumentalization process was relatively short.
- The didactic proposal presented in the tool sometimes did not promote the learning of concepts. In both cases, the teacher provided worksheets.

Potentials of the tool

- The instrumentation process was short due to the high degree of the structuring of concepts presentation.

Alberto - The 1st video explains well the definition of concavity of a graph of a function at a given interval. You understand Elisabete?

Elisabete - Yes. The graph has a concavity facing upwards if in that range is above all lines tangent to the curve. Is this correct what I said?

Alberto - Okay. Now come to the 2nd and 3rd videos. The signal of a , the coefficient of the term of degree 2, the quadratic function, varies the direction of the concavity of the parabola. Correct?

Elisabete - Yeah, I get it. And the greater the absolute value of a the parabola more approaches the ordinate axis.

Concavidade do gráfico de uma função quadrática

Será que o sinal de a está relacionado com o sentido de concavidade da função? Faz variar a e verifica o que acontece.

$a = 0.6$

resumo

conclusão

Potentials of the tool

- Learning concepts is highly potentiated by the possibility of manipulate different representations of the concept (eg. the manipulation of selectors).

Elisabete - The 1st video consolidates what we learned earlier: the parameter a will cause the parabola stretch or shrink horizontally.

Alberto - Explain yourself better!

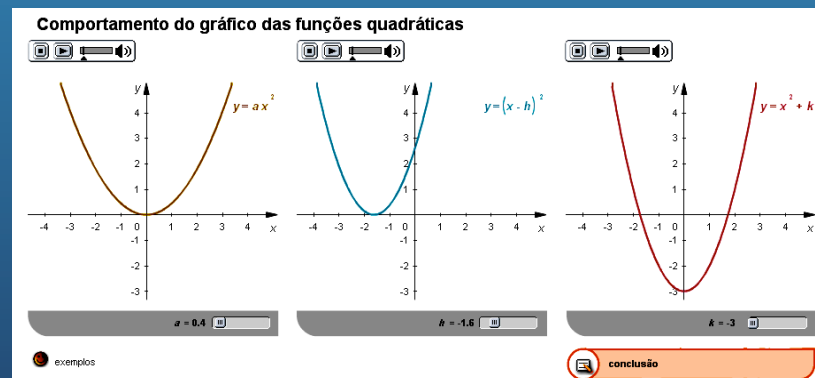
Elisabete - Notice that the longer the absolute value of a more parabola closes around the y-axis.

Alberto - Okay, let's now see the h in $(x-h)^2$?

Elisabete - The parabola moves horizontally to the right and to the left, when we change h .

Alberto - And it is the x-coordinate of the vertex of the parabola.

Elisabete - I get it! And the k does the parabola go up or down. And is the ordinate of the vertex.



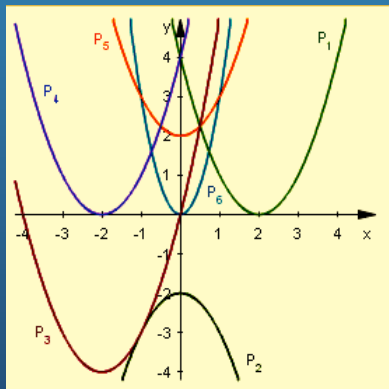
Potentials of the tool

- Motivation increased with the performance of self-corrective tasks providing competition among students.
- Self-corrective tasks improve performance of algebraic procedures even when these are routine.

(Each of the students went to a computer and individually used the CD-ROM to solve the task, proposed by the teacher and based on the exercises of the tool).

Elisabete - Of the 36 possible correct answers I missed 4. I did not hit, for example, in the calculation of the zeros and the coordinates of the vertex of the parabola. But I returned to do this exercise (on the sheet of paper) and found where I went wrong.

Alberto - I got it all at first. Good!



Função	Zeros	Positiva	Decrescente	Coordenadas do vértice	\emptyset
$f(x) = 3x^2$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	$\mathbb{R} \setminus \{-2\}$
$g(x) = x^2 + 2$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	$\mathbb{R} \setminus \{2\}$
$h(x) = -x^2 - 2$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	$] -\infty, 0 [$
$i(x) = (x-2)^2$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	$] -\infty, 2 [$
$j(x) = (x+2)^2$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	$] -\infty, -2 [$
$k(x) = (x+2)^2 - 4$	<input type="text"/> ; <input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/> , <input type="text"/>	\mathbb{R}
					$] 0, +\infty [$
					$\mathbb{R} \setminus \{0\}$
					$\mathbb{R} \setminus [-4, 0]$

Constraints of the tool

- Formal language used in audio and video texts is hard to grasp.

Alberto - Did you understand the resolution of this problem?

Elisabete - I had difficulty in calculating the values that the length of the rectangle can take.

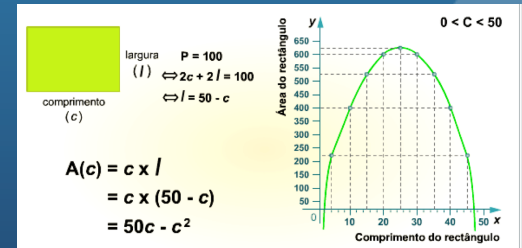
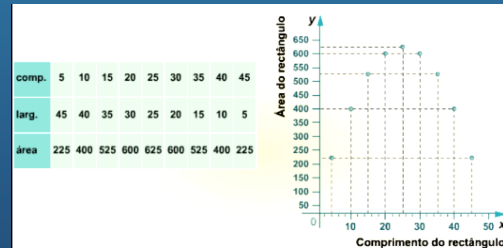
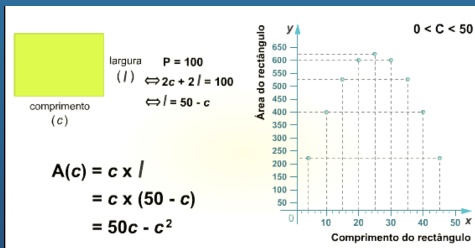
Alberto - Would you repeat the video?

Elisabete - Yes.[after 2nd visualization of the video] What values can c take? Explain to me this step because the video is not very clear on this point.

Alberto - Consider: the width $l=50-c$ can not be negative or zero. $50-c>0 \Leftrightarrow -c>-50 \Leftrightarrow c<50$.

Elisabete - And c is grather than zero, it is a lenght. And the value of c which corresponds to a maximum area? How do you find this value without having heard the answer given by the video? Did you notice that the parabola drawn on the CD-ROM does not pass the origin? It is wrong because when $c=0$ the area is zero. We found an error in the CD-ROM!

What is the rectangle of largest area that can be constructed with a cord of 1 meter?



Constraints of the tool

- The use of symbolic representations that students sometimes did not dominate because they are too formal;

Teacher: Let's see. You are not understand?

Vanessa: No, I'm not realizing.

Teacher: You do not understand what?

Vanessa: I do not understand this thing of signals.

[They hear a little more of the audio of negation]. (...)

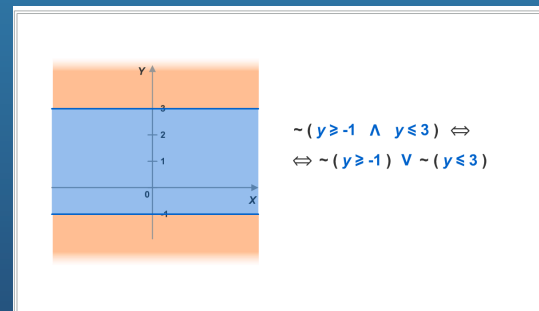
Teacher: So what is the question?

Vanessa: It's here “*stor*”, I can not understand these expressions.
[Points to the expressions below]

$$A = \{(x,y) \in \mathbb{R}^2 : y \leq 2\} \quad \bar{A} = \{(x,y) \in \mathbb{R}^2 : y > 2\}$$

$$\sim (y \leq 2) \quad \Leftrightarrow \quad y > 2$$

$$\sim (y > 2) \quad \Leftrightarrow \quad y \leq 2$$



Leis de De Morgan Definição

Estas são as Primeiras Leis de De Morgan, que podem ser enunciadas da seguinte forma:

- a **negação da conjunção** de duas condições é equivalente à **disjunção da negação** de cada uma delas.
- a **negação da disjunção** de duas condições é equivalente à **conjunção da negação** de cada uma delas.

Simbolicamente:

$$\sim (a \wedge b) \quad \Leftrightarrow \quad \sim a \vee \sim b$$

$$\sim (a \vee b) \quad \Leftrightarrow \quad \sim a \wedge \sim b$$

Constraints of the tool

- Solving self-corrective tasks can be performed without an understanding of the mathematical concepts involved.

Students can use a process of trial and error, correcting their wrong answers.

- The tool is not *open-access* which restricts its use outside the context of the classroom.

These constraints lead teachers to develop specific kinds of documents.

Considera o ponto P de coordenadas $(d + 1, d - 1)$. Determina d de modo a que P pertença:

a) à bissetriz dos quadrantes pares. $d =$

b) à reta de equação $x = -2$. $d =$

c) à região do plano $y < 2$. $d <$

d) ao eixo Ox. $d =$

e) ao 3º quadrante. $d <$

Avaliação ✕

A fazer	5
<input type="radio"/> Corretas	1
<input type="radio"/> Erradas	2
TOTAL	20 %

Soluções

Apagar erradas

Recomeçar

Final remarks

- Highly structured technological tools (akin to tutorials) can be good learning tools.
- The process of instrumental genesis can be short when the tool is very structured, but can cause understanding difficulties for less gifted students.
- When computational tools are used, documentational genesis can become a powerful artefact to develop schemes that promote student's learning.
- The documentational genesis can be developed either based on the potentialities either based on the constraints of the tool.

Thank you for your attention