

Using technology for maths teaching
and learning:
Instructional design, digital books
and automated feedback

Dr. Christian Bokhove

Shinshu University – 1 March 2017

Contents

- Some general comments about technology use
- Does technology work for maths education
 - Yes, but under certain conditions
 - Integrate in ‘digital books’
- Features of those digital books
- Examples of technology
 - I have added many, I will not cover all of them.

Who am I

- Dr. Christian Bokhove
- From 1998-2012 teacher maths, computer science, head of ICT secondary school Netherlands
- PhD 'Use of ICT for acquiring, practicing and assessing algebraic expertise' with Prof. Van Maanen and Prof. Drijvers
- Since 2012 Lecturer at University of Southampton
 - Maths education
 - Technology use
 - Large-scale assessment



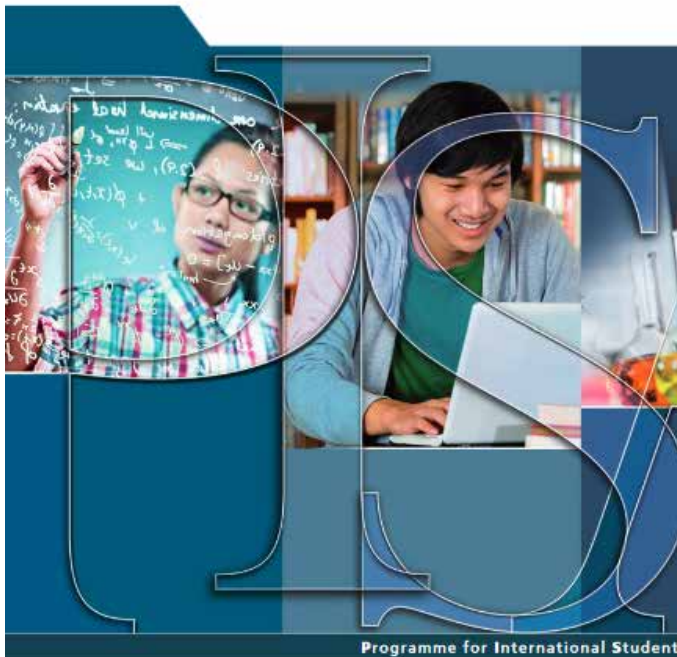
INTRODUCTION

There is a lot to do with technology



Students, Computers and Learning

MAKING THE CONNECTION



Programme for International Student Assessment

On average, in the past 10 years there has been no appreciable improvement in student achievement in reading, mathematics or science in the countries that have invested heavily in information and communication technologies for education.



In Australia, New Zealand and the United Kingdom, every 15-year-old has individual access to a computer at school.

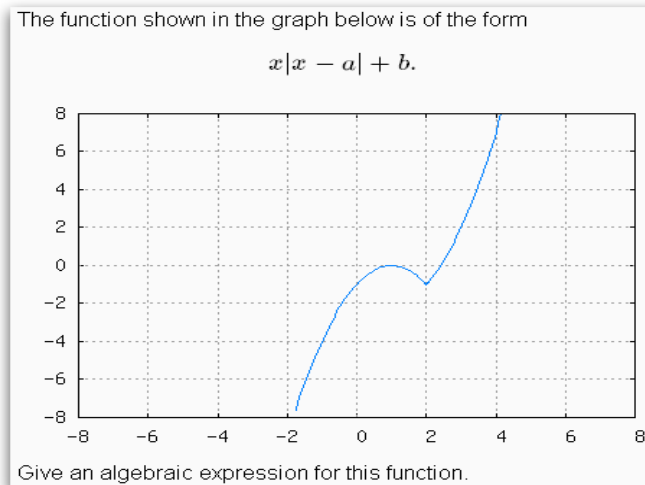
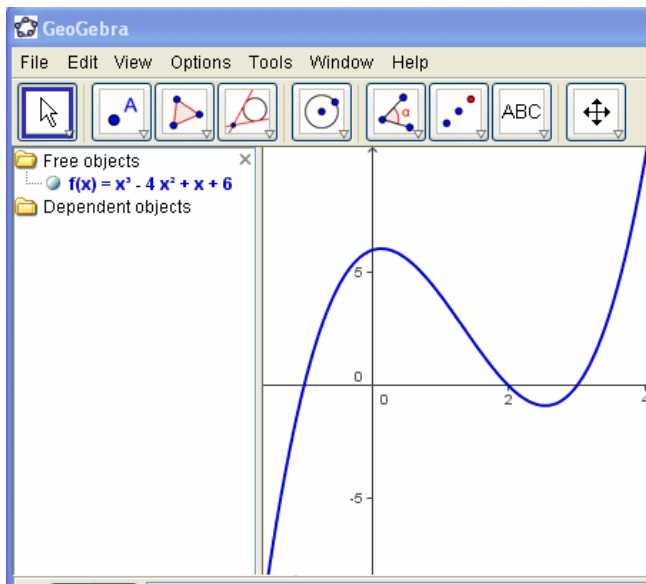


In Germany, Italy and Japan, there is **only one school computer** available for every four 15-year-old students.



In mathematics education...

- 17th ICMI study “Mathematics Education and Technology-Rethinking the Terrain”
- 22nd ICMI study “Task Design In Mathematics Education”
- ...



<http://timss2015.org/>

TIMSS & PIRLS



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TIMSS 2015 AND TIMSS ADVANCED 2015 INTERNATIONAL RESULTS

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[TIMSS ADVANCED 2015](#)

PRESS RELEASE

Chestnut Hill, Mass. (11/29/2016) – Singapore, Hong Kong SAR, Korea, Chinese Taipei, and Japan continue outperforming all participating countries in mathematics at the fourth and eighth grades, maintaining a 20 year edge according to results released today from TIMSS, the longest running, large scale international assessment of mathematics and science education in the world.

[Read more](#)

Highlights from TIMSS 2015



Exhibit 9.6: Computer Activities During Mathematics Lessons

Reported by Teachers

Country	Computers Available for Students to Use in Mathematics Lessons			Percent of Students Whose Teachers Have Them Use Computers at Least Monthly			
	Percent of Students	Average Achievement		To Explore Mathematics Principles and Concepts	To Practice Skills and Procedures	To Look Up Ideas and Information	To Process and Analyze Data
	Yes	Yes	No				
Sweden	65 (3.6)	499 (4.0)	502 (4.0)	25 (3.7)	38 (4.0)	32 (4.2)	26 (3.9)
Australia	62 (3.4)	512 (3.5)	506 (5.4)	51 (3.5)	52 (3.6)	48 (3.6)	44 (3.2)
Kazakhstan	53 (3.9)	531 (7.6)	525 (7.4)	45 (4.5)	50 (4.1)	51 (4.2)	45 (4.5)
Canada	50 (3.3)	528 (3.7)	533 (3.2)	35 (2.8)	36 (3.1)	33 (3.0)	31 (3.1)
Chile	49 (4.6)	423 (5.5)	437 (5.8)	29 (4.3)	36 (4.4)	32 (4.5)	36 (4.3)
Egypt	48 (3.9)	395 (6.1)	390 (5.8)	35 (4.0)	42 (4.1)	45 (4.0)	32 (3.7)
Russian Federation	47 (3.5)	535 (5.1)	540 (6.4)	36 (3.5)	41 (3.6)	42 (3.2)	34 (3.5)
New Zealand	47 (3.5)	501 (4.8)	488 (5.7)	36 (3.3)	35 (3.3)	35 (3.3)	33 (3.5)
United Arab Emirates	44 (2.2)	481 (4.5)	456 (3.8)	38 (2.0)	40 (2.1)	40 (2.2)	37 (2.3)
Japan	43 (3.7)	585 (4.1)	588 (3.4)	3 (1.0)	6 (1.8)	4 (1.3)	5 (1.5)
Italy	43 (3.7)	493 (4.3)	495 (4.1)	28 (3.2)	29 (3.3)	31 (3.5)	26 (2.9)
Norway (9)	40 (3.9)	513 (3.5)	513 (3.2)	27 (3.9)	35 (4.1)	27 (4.0)	29 (3.8)
Jordan	39 (3.3)	394 (6.5)	378 (4.0)	29 (3.4)	28 (3.4)	32 (3.5)	25 (3.3)
Thailand	39 (4.5)	442 (8.5)	425 (6.1)	25 (4.0)	26 (4.2)	28 (4.3)	23 (4.1)
United States	39 (2.9)	519 (5.0)	518 (4.3)	r 27 (2.8)	r 31 (2.9)	r 29 (2.8)	r 26 (2.8)
Korea, Rep. of	39 (3.6)	604 (4.3)	607 (3.6)	25 (3.3)	22 (3.1)	24 (3.2)	19 (2.6)
Lithuania	38 (4.0)	508 (4.9)	512 (4.5)	21 (3.7)	24 (3.4)	29 (3.8)	23 (3.5)
Georgia	38 (3.6)	453 (6.6)	452 (4.5)	33 (3.8)	31 (3.5)	34 (3.8)	33 (3.7)
Qatar	36 (2.6)	422 (6.6)	445 (4.3)	31 (2.3)	33 (2.5)	30 (2.7)	26 (2.9)
Singapore	35 (2.5)	617 (6.0)	621 (4.1)	27 (2.2)	27 (2.3)	23 (2.0)	19 (2.0)
Hungary	30 (3.8)	509 (8.0)	516 (4.6)	20 (3.3)	27 (3.6)	22 (3.2)	18 (3.0)
Bahrain	30 (2.8)	458 (3.8)	452 (2.2)	23 (2.4)	23 (2.7)	24 (2.8)	16 (1.9)
England	29 (4.1)	511 (9.7)	520 (6.0)	17 (3.6)	23 (3.7)	17 (3.3)	13 (2.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2015



Exhibit 9.7: Student Use of Internet for Schoolwork

Reported by Students

Country	Percent of Students Who Use the Internet to Do the Following Tasks				
	Access the Textbook or Other Course Materials	Access Assignments Posted Online by the Teacher	Collaborate with Classmates on Assignments or Projects	Communicate with the Teacher	Find Information, Articles, or Tutorials to Aid in Understanding Mathematics
Australia	55 (1.4)	66 (1.2)	63 (0.8)	46 (1.1)	57 (1.0)
Bahrain	56 (1.0)	43 (1.1)	77 (0.8)	41 (1.1)	58 (0.8)
Botswana (9)	46 (0.8)	37 (1.0)	58 (1.1)	36 (0.8)	54 (0.8)
Canada	45 (1.5)	58 (2.0)	76 (1.0)	32 (1.2)	56 (1.2)
Chile	62 (1.0)	37 (1.4)	79 (0.9)	25 (1.2)	60 (1.0)
Chinese Taipei	74 (0.9)	50 (1.1)	72 (1.0)	28 (1.0)	38 (0.8)
Egypt	57 (1.1)	34 (1.0)	58 (1.0)	56 (1.2)	64 (1.0)
England	54 (1.5)	71 (1.4)	53 (1.4)	33 (1.9)	66 (1.1)
Georgia	76 (1.3)	44 (1.5)	73 (1.3)	31 (1.4)	47 (1.2)
Hong Kong SAR	51 (1.3)	64 (1.9)	76 (1.3)	33 (1.2)	61 (1.1)
Hungary	40 (1.1)	58 (1.2)	76 (1.1)	26 (1.3)	41 (1.1)
Iran, Islamic Rep. of	60 (1.4)	40 (1.1)	56 (1.2)	31 (1.0)	52 (1.2)
Ireland	34 (1.2)	35 (2.6)	50 (1.2)	12 (1.2)	44 (1.0)
Israel	64 (1.2)	68 (1.4)	60 (1.2)	32 (1.3)	55 (0.9)
Italy	50 (1.1)	34 (2.1)	75 (1.1)	27 (1.5)	41 (1.0)
Japan	23 (0.8)	16 (0.9)	28 (1.0)	5 (0.5)	30 (0.8)
Jordan	65 (1.1)	42 (1.2)	70 (1.3)	49 (1.2)	61 (1.0)
Kazakhstan	65 (1.1)	39 (1.5)	76 (0.9)	24 (1.3)	66 (0.9)
Korea, Rep. of	51 (1.0)	43 (1.3)	69 (1.1)	13 (0.7)	45 (0.9)
Kuwait	x x	x x	x x	x x	x x

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2015



- So there are big differences between countries...
- It is not clear when technology contributes positively
- My research in the last years has focussed on finding out a bit more...
- I will give some examples of ways that seem positive.

In 2010 I looked at specific criteria for algebra software

Table 2 The top five of most important and bottom five of least important criteria

Rank	Description	Weight
1	The stability and performance of the tool	4.89
2	The tool is easy to use for a student (e.g. equation editor, short learning curve, interface)	4.85
3	The tool is able to display formulas correctly	4.81
4	The tool is mathematically sound and faithful to the underlying mathematical properties (e.g. conventional representations, sound operations)	4.74
5	The tool stores the answers given by a student	4.70
23	The tool has the ability to randomize algebra assignments	3.96
24	The cost of the tool	3.74
25	The tool makes use of standards (e.g. QTI, SCORM)	3.72
26	The tool enables the student to use a computer algebra system as a tool	3.63
27	The licensing of the tool (e.g. open, proprietary)	3.41

Bokhove, C., & Drijvers, P. (2010). Digital tools for algebra education: criteria and evaluation. *International Journal of Computers for Mathematical Learning*, 15(1), 45-62.

What should technology look like

This has developed over the years:

- From distinct, separate tools to integrated
- From offline towards online
- From not storing data to storing data

It became more and more clear that it made sense to make an INTEGRATED design: digital books.

INTEGRATED DESIGN: DIGITAL BOOKS

International Conference on Mathematics Textbook Research and Development 2014. Strand on e-Textbooks and technology. From Prof. Yerushalmy's talk

e-Textbook

Necessary Dimensions of Design

- **Interactive** engagements for students and teachers
interactions amongst learners and between learner and the textbook
- **Flexible Integration**
integration of 'adds-on' traditionally not part of textbooks
input/output from and to external systems
- **Evolving** continuously by its authors/users
interactions amongst authors and users

Examples

- Apple iBooks
- Great looks
- Limited interaction
- Limited student management options

The screenshot shows the iTunes Preview page for the book "Learn Pythagorean Theorem Through Exploration" by Kyle Pearce. The page is displayed in a browser window with a navigation bar at the top containing links for Store, Mac, iPod, iPhone, iPad, iTunes, and Support. The main content area features the book title, author name, and a brief description. A "View in iBooks" button is visible below the book cover. The "Description" section provides more details about the book's content. A "Screenshots" section shows a preview of the book's content, including a diagram of a right-angled triangle with sides labeled a , b , and c , and the equation $a^2 + b^2 = c^2$. The diagram shows a right-angled triangle with a vertical side of length a , a horizontal side of length b , and a hypotenuse of length c . The area of the square on side a is labeled a^2 , the area of the square on side b is labeled b^2 , and the area of the square on the hypotenuse c is labeled c^2 . The diagram illustrates the Pythagorean Theorem by showing that the sum of the areas of the two smaller squares is equal to the area of the largest square.

Towards digital textbooks

- Digital textbook: theory, examples, explanations
- Interactive content (in widgets)
- Interactive quizzes (formative assessment, feedback)
- Integrated workbook

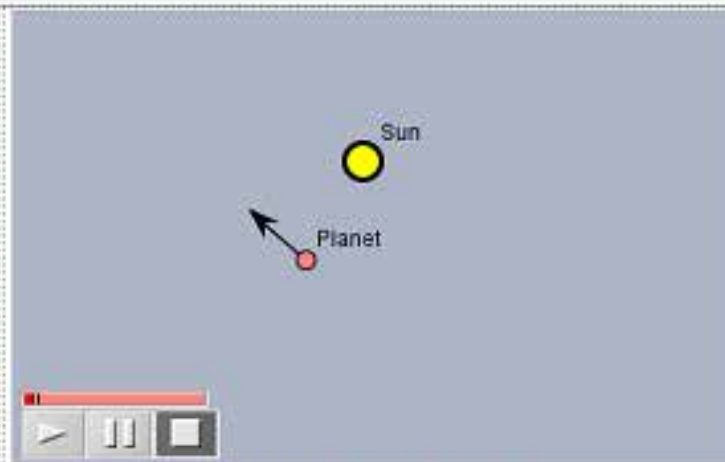
- CBooks
- My profile
- Class managem...
- Workspaces...
- Features schooladmin
 - School users
 - School classes
 - School configura...

CBooks

Text: Title:

Orbital mechanics

To the right you can see a space with one solitary sun and a planet positioned close to it, and with an initial velocity. You are going to see what happens with a planet when you vary its position and initial velocity. Draw what trajectory you think the planet will make below, then start the tool on the right to check your suspicion.



What can you say about the trajectory and the speed?

 problems

Options

Practice

1 2 3 4 5 6 7 8 9 10 11 12 13

Variables for random parameters:

More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. Storing results
4. Instructional design
5. Authorability
6. Interoperability and standards

More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets

2. Interaction (feedback)

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4. Instructional design

5. Authorability

6. Interoperability and standards

CBooks

My profile

Class managem...

Workspaces...

Features schooladmin

School users

School classes

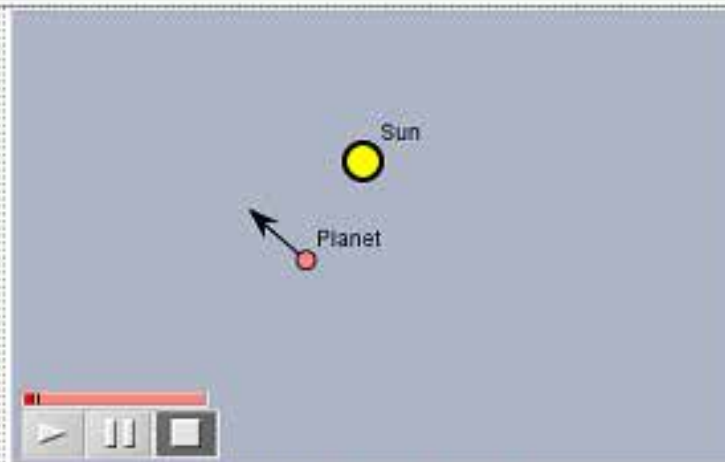
School configura...

CBooks

Text: Title:

Orbital mechanics

To the right you can see a space with one solitary sun and a planet positioned close to it, and with an initial velocity. You are going to see what happens with a planet when you vary its position and initial velocity. Draw what trajectory you think the planet will make below, then start the tool on the right to check your suspicion.



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Options

Practice

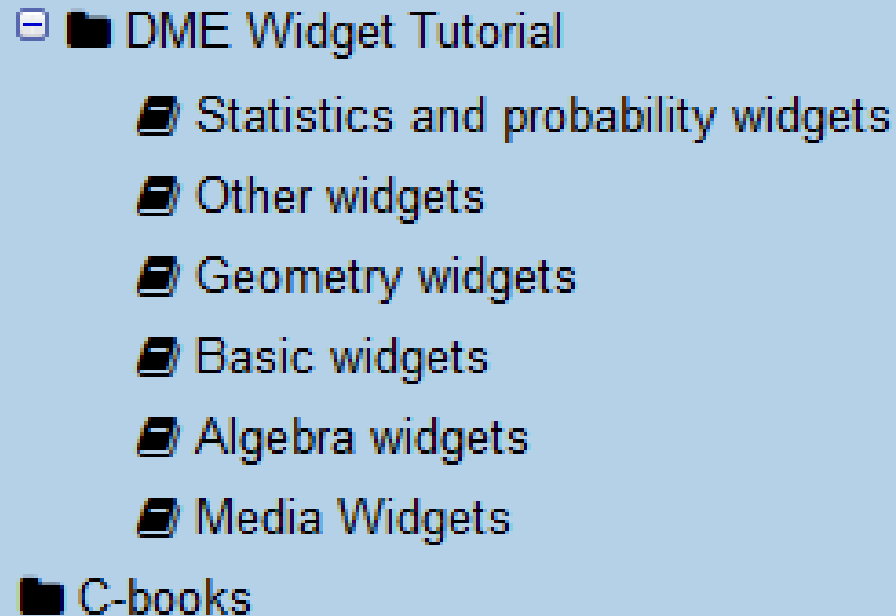
1 2 3 4 5 6 7 8 9 10 11 12 13

Variables for random parameters:

- Purpose of technology
 - e.g. tension learning and doing (e.g. Ainley & Pratt, 2002)
 - Use to learn, learn to use
- Openness
 - Beeson (1998), open and closed
 - Bliss & Ogborn (1998), exploratory and expressive
 - Buchberger (1990), black box, white box, glass box
- Interactivity

DEMO WIDGETS

- Browse some widgets
- <http://mc2dme.appspot.com/>



Widget example: equations

Digitale Wiskunde Omgeving Freudenthal Instituut

VO-HO testschool
amidocent amidocent

>> Algebra met Inzicht deel 1 t/m 4 REVISIE

1. Algebra met Inzicht: deel 1

Uitloggen

Opgave 1.7

Los de volgende vergelijking op:

voorbeeld

$$(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)$$
$$x^2 - 7x + 12 = 0 \text{ of } 8x - 11 = 3x + 14$$
$$x^2 - 7x + 12 = 0 \text{ of } 5x - 25 = 0$$
$$x^2 - 7x + 12 = 0 \text{ of } 5x = 25$$
$$x^2 - 7x + 12 = 0 \text{ of } x = 5$$

Serie 1 Score: 0
 Serie 2 Score: 10

Opdracht: **1** 2 3 4 5 6 7

Score: 10

Basic Widgets

General Properties

Score:	Points:
<input checked="" type="checkbox"/> Equivalent	<input type="text" value="0"/>
<input type="checkbox"/> Form	
<input checked="" type="checkbox"/> Answer needed	<input type="text" value="10"/>
<input type="checkbox"/> Exact	

Stepwise Formula Answer Box

$3x^2 + 2x + x^2 - x + 3 + 4x - 2 =$
 $3x^2 + x^2 + 2x - x + 4x + 3 - 2 =$
 $4x^2 + 5x + 1$

✓

Small Formula Answer Box

$$5 \cdot 3 = \text{[input box]}$$

Text Answer Box

f(x) [calculator icon] [math symbols icon]

[input box]

Randomized variables

$$\#a\# \cdot \#b\# \cdot x^{\#p\#}$$

$$\#a \cdot \#b\# \cdot x^{\#p\#}$$

$$\#a \cdot \#b\# \cdot \#x^p\#$$

$$\#a \cdot \#b \cdot x^p\#$$

Stepwise Equation Answer Box

$\frac{1}{4}x + 7 = 5x - 2$

$3x + 28 = 20x - 8$ $\left. \begin{array}{l} \times 4 \\ - 3x \\ + 8 \end{array} \right\}$

$28 = 17x - 8$

$36 = 17x$

Small Equation Answer Box

Solve the equation: $x^2 - 3x + 2 = 0$.

[input box]

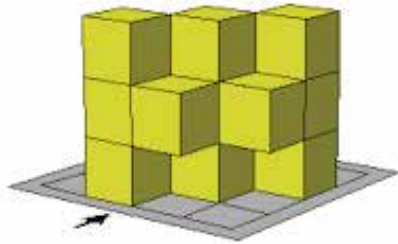
Check Text Answer Box

Which day comes after Sunday?

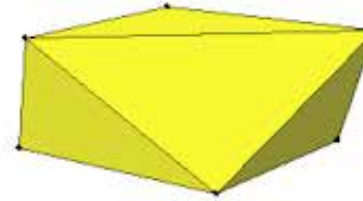
[input box]

Geometry Widgets

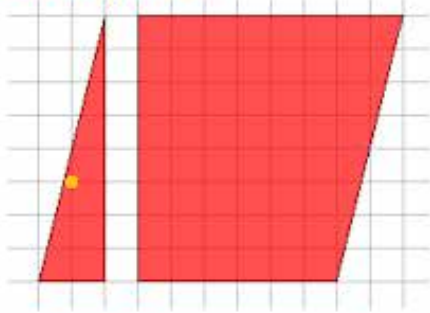
Building Blocks



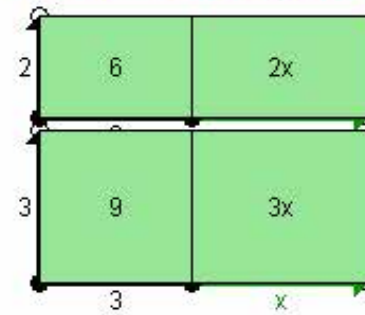
3D Construction tool



Cutting Areas



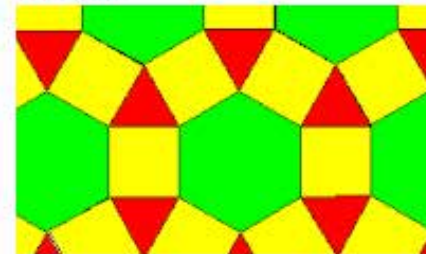
2D Geometrical Algebra



Drawing in Space



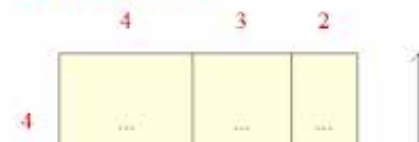
Mozaik



Tilings

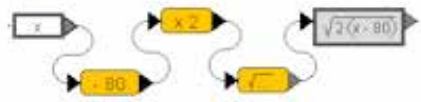


Area Algebra

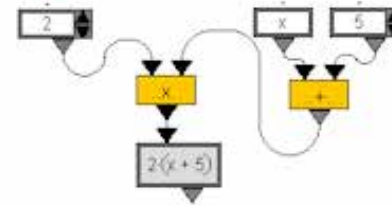


Algebra Widgets

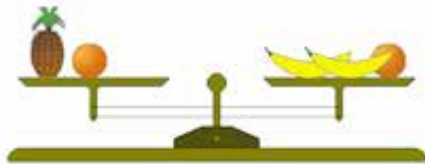
Algebra Arrows



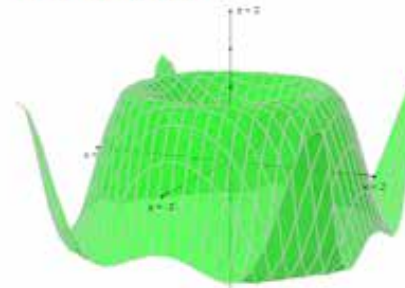
Algebra Trees



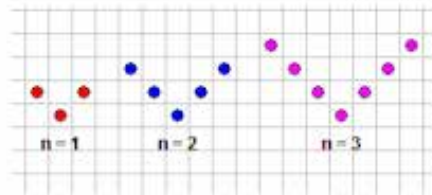
(Fruit) Balance



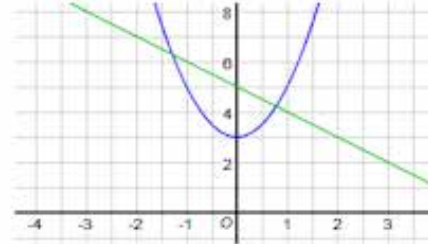
3D Graphing



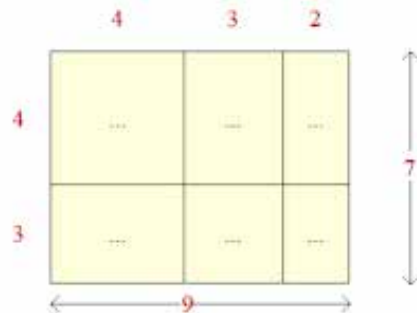
Patterns with Dots



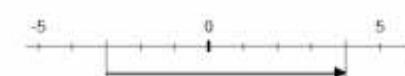
Graph Tool



Area Algebra

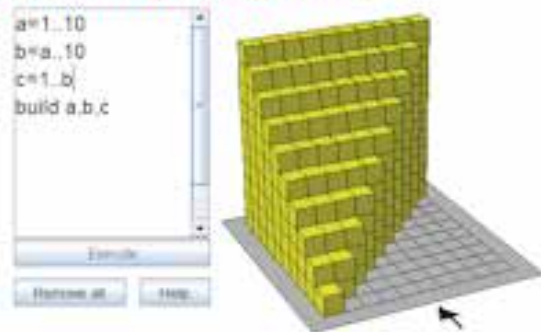


Number Line



Other Widgets

Building Programs



C-Book Units

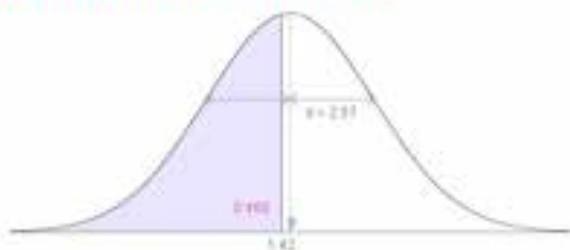
1. Building Programs
2. Note

Note



Statistics and Probability Widgets

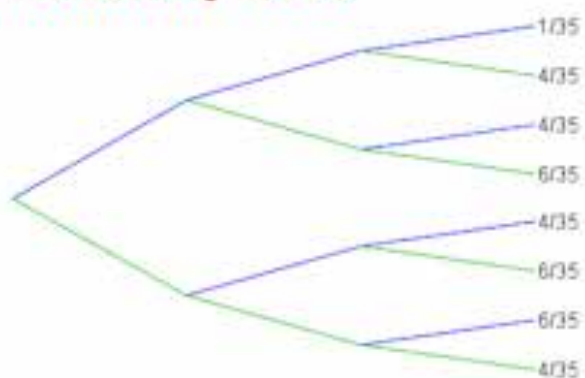
Normal Distribution



Flow Diagrams



Probability Trees

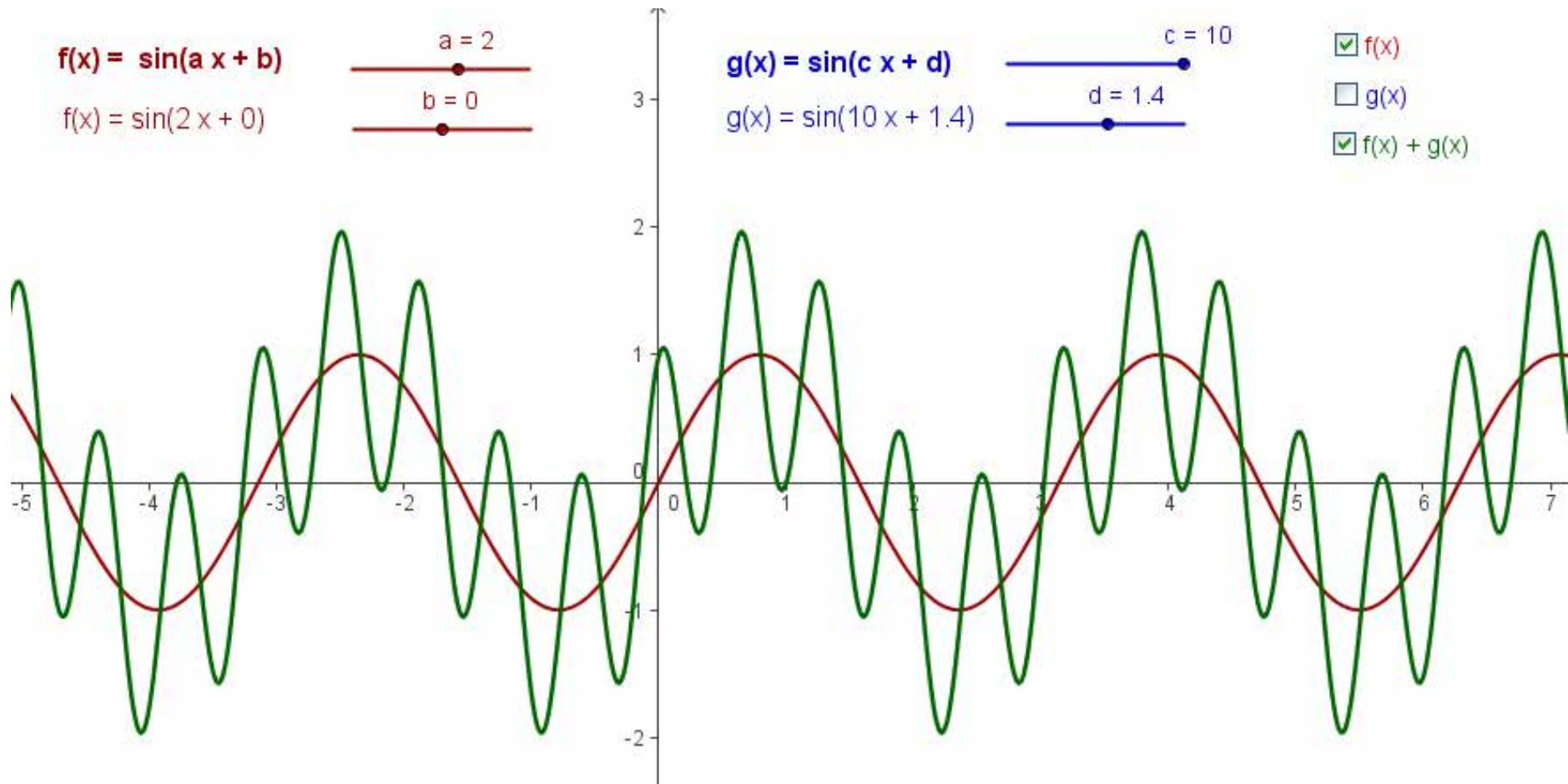


What could 'open' widgets add?

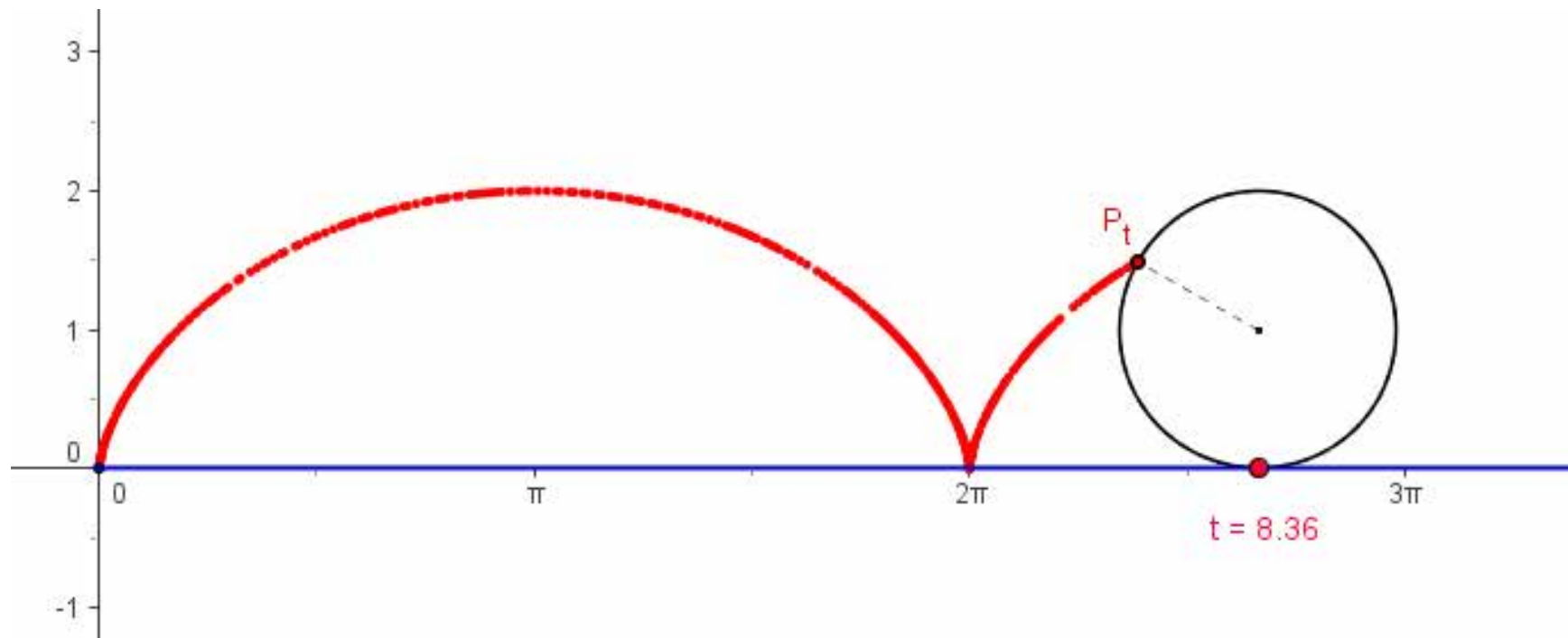
- **Visualizing**
students can “see” abstract concepts
- **Representations**
students can make connections
- **Experiments**
students can discover mathematics

Example: geogebra

Visualizing Parameters



Example: geogebra Visualizing Movement



Multiple Representations

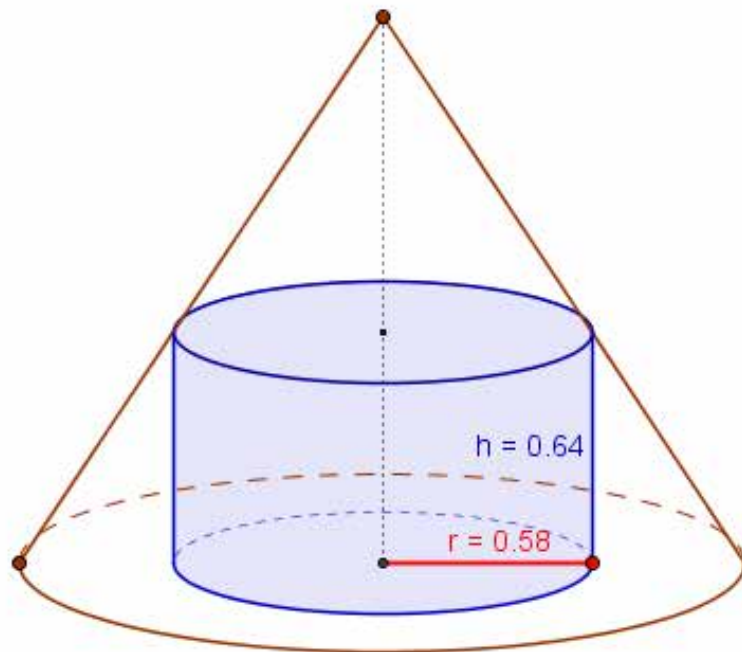
- GeoGebra: Bidirectional Connection
Symbolic \Leftrightarrow Graphics
- Examples
 - Coordinates \Leftrightarrow Point in Coordinate System
 - Circle's Equation \Leftrightarrow
Circle drawn in Coordinate System
 - Function's Equation \Leftrightarrow Graph of Function

Example: geogebra

Representations: Sketch & Graph

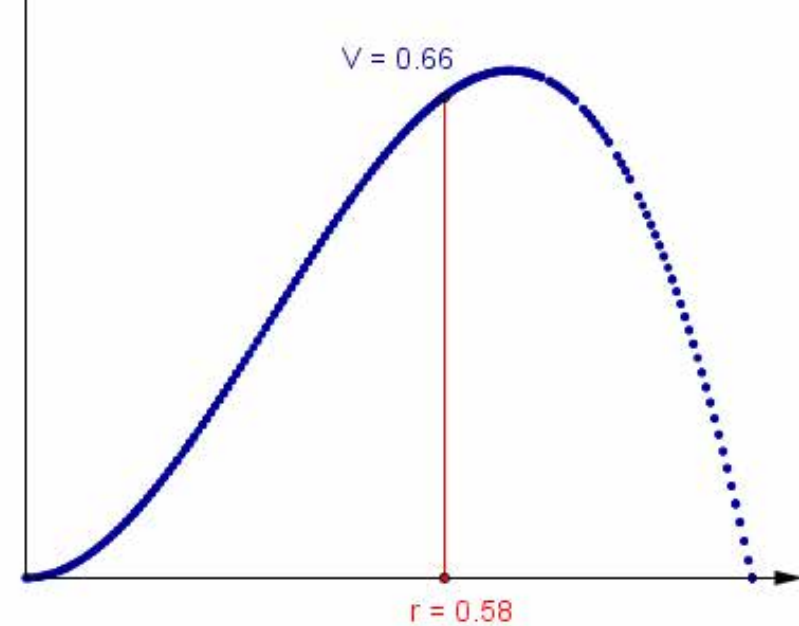
Cone

Height = 1.5
Diameter = 2



Cylinder - Volume

$$V = r^2 \pi h$$



More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
- 2. Interaction (feedback)**
3. Storing results
4. Instructional design
5. Authorability
6. Interoperability and standards

Feedback

- Essential part formative assessment (Black & William, 1998)
- But depends on feedback type(s) e.g. task, process, self-regulation, self (Hattie & Timperley, 2007)
 - Task e.g. ‘that is the wrong answer’
 - Process e.g. ‘have another look at what you did’
 - Self-regulation e.g. ‘review your answers and decide what topic you want to revise’
 - Self e.g. ‘well done!’
- Issues
 - Positive/negative feedback
 - Timing

Feedback in computer systems

Review Vander Kleij et al. (2015)

- Types
 - Elaborated feedback 0.49
 - Providing the correct answer 0.32
 - Correctness of answer 0.05
- Higher order learning even more, as well as for mathematics compared to social sciences

Example of feedback: embedded

Solving equations 3

Exercise 4

Solve the equations below. Press enter to check each step.

a. $\sqrt{\quad}$ \square^{\square} \square^2 $\frac{\square}{\square}$ (\square) more  


$$x^2 - 4 = 3x$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4$$




Solution(s) are missing. 
Add the missing one(s).


Example of feedback: custom feedback

Make as many expressions that are equivalent with 35. Use the button   to get a formula editor.


35  

Putting in the number. Not very creative is it? 


$34 + 1$  

What about a different operation than addition? 


$33 + 2$  

What about a different operation than addition? 

$350/10 =$

What about a different operation than division? 

$35 \cdot \sin(1/2 \cdot \pi)$

You used sin or cos. That's pretty creative! 

$\int_0^{\sqrt{70}} x \, dx$

You used an integral. That's pretty creative! 

Example of feedback: rule-based feedback

$\sqrt{\square}$ \square^{\square} \square^2 $\frac{\square}{\square}$ (\square) more hint help step solve ↓ ↑

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

factorize the three expressions

product=0, equate factors to 0

The diagram illustrates the process of solving a quadratic equation. It starts with the equation $x^2 - 4x + 3 = 0$. An arrow points to a yellow box containing the instruction "factorize the three expressions". A second arrow points to another yellow box containing the instruction "product=0, equate factors to 0". Finally, the solution is given as $x = 1$ or $x = 3$.

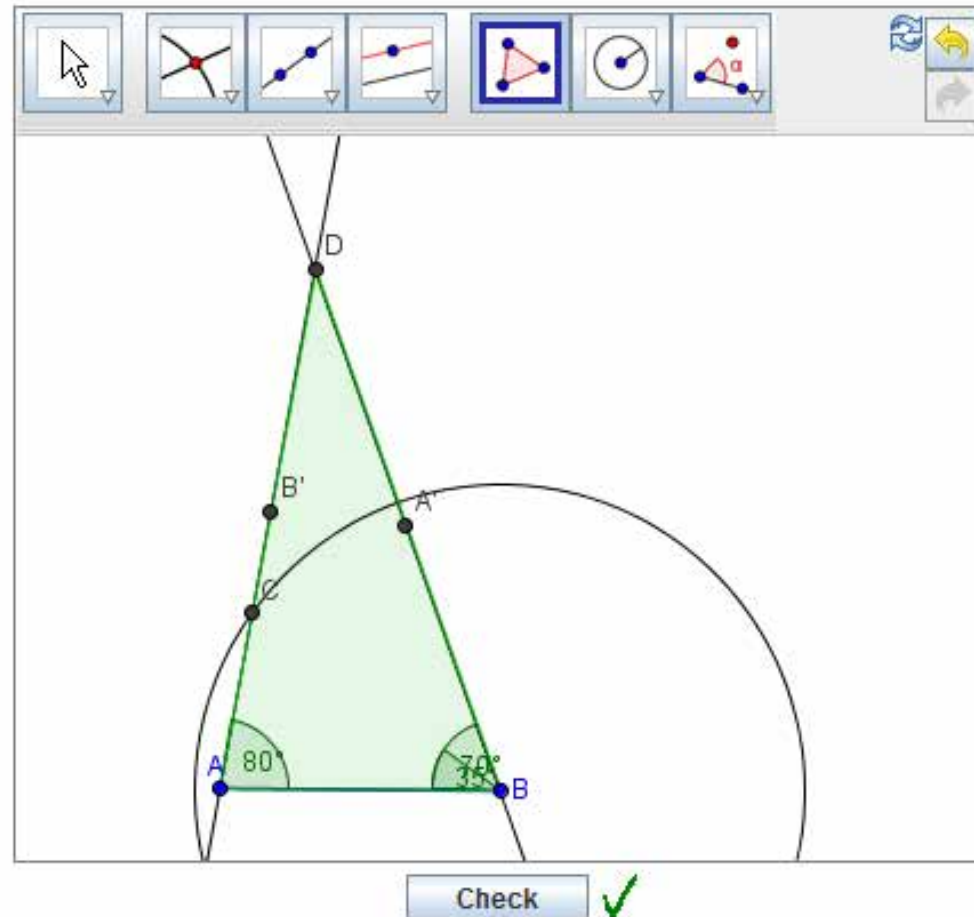
Also geometry checking

Find the triangle

The figure shows a segment AB . Construct a triangle ABC with the following properties:

- $\angle A = 80^\circ$
- The segment of the bisector from B within the triangle has length 4.
- C is above AB .

Don't forget to draw the requested triangle with the Polygon option!



More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
- 3. Storing results**
4. Instructional design
5. Authorability
6. Interoperability and standards

Storing results

- Imagine 10 classes with 30 students and numerous tasks: management system
- But diversity of data types, ranging from simple checkbox to complexity of geometry widget
- Useful for Learning Analytics and studying misconceptions

Store student results, and use these as a teacher to study misconceptions and for starting classroom discussions

DWO Math Enviroment - Microsoft Internet Explorer wordt aangeboden door St. Michael College

http://www.fi.uu.nl/dwo/voho/frameset.html

Digitale Wiskunde Omgeving Freudenthal Instituut

VO-HO testschoon
amidocent amido

Resultaten Uitloggen

Overzicht modules
Mijn Profiel
Klassen beheren
Modules beheren

Resultaten van klas:
- rsg_enkhui

Copy

Module "Algebra met Inzicht deel 1 t/m 4"

Klas rsg_enkhui Activ. 1 Activ. 2 Activ. 3 Activ. 4

	8 % (in 4 min)		9 % (in 36 min)	0 % (in 37 sec)
	75 % (in 86 min)	100 % (in 66 min)	98 % (in 74 min)	14 % (in 114 min)
	83 % (in 87 min)	80 % (in 11 min)	93 % (in 50 min)	94 % (in 40 min)
	75 % (in 59 min)	100 % (in 19 min)	99 % (in 49 min)	85 % (in 53 min)
	75 % (in 77 min)	100 % (in 20 min)	94 % (in 37 min)	41 % (in 50 min)
	80 % (in 71 min)	100 % (in 12 min)	100 % (in 30 min)	28 % (in 18 min)
	60 % (in 55 min)	100 % (in 8 min)	93 % (in 19 min)	14 % (in 44 min)
	66 % (in 112 min)	0 % (in 6 sec)	36 % (in 14 min)	

More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
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5. Authorability
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- (i) students learn a lot from what goes wrong,
- (ii) but students will not always overcome these if no feedback is provided, and
- (iii) that too much of a dependency on feedback needs to be avoided, as summative assessment typically does not provide feedback.

These three challenges are addressed by principles for **crises**, **feedback** and **fading**, respectively.

Crisis-tasks

“students learn a lot from what goes wrong”

The screenshot shows a math software interface with a toolbar at the top containing icons for square root, powers, fractions, and a 'meer' dropdown. The main workspace contains the following steps:

$$(x^2 + 3x - 3) \cdot (8x - 6) = (x^2 + 3x - 3) \cdot (4x + 12)$$
$$8x^3 + 18x^2 - 42x + 18 = 4x^3 + 24x^2 + 24x - 36$$
$$4x^3 - 6x^2 - 66x = -54$$
$$4x \left(x^2 - 1\frac{1}{2}x - 16\frac{1}{2} \right) = -54$$

A yellow checkmark is visible to the left of the final equation. On the right side, four curved arrows point downwards from the first equation to the second, second to third, third to fourth, and fourth to a feedback box. The feedback box is yellow and contains the text: "Je ben goed aan het herschrijven." with a close button (X) in the top right corner.



“Failure is, in a sense, the highway to success” - Keats

Feedback: overcoming a crisis

“but students will not always overcome these if no feedback is provided”

The screenshot shows a math problem-solving interface. At the top, there are navigation icons and a search bar. The main content area displays the equation $(2x^2 + 4x - 3) \cdot (8x - 3) = (2x^2 + 4x - 3) \cdot (3x + 12)$. Below this, the student has written $2x^2 + 4x - 3 = 0$ of $8x - 3 = 3x + 12$. The solution $x = 3$ is marked with a yellow checkmark. A dashed line points from a feedback message to the equation. The feedback message is a yellow box with a close button (X) and the text: "Je dreigt twee oplossingen kwijt te raken. Bedenk dat de expressie $(2x^2 + 4x - 3)$ ook twee oplossingen oplevert. Vul aan."

You are about to lose two solutions. Keep in mind that the expression $(2x^2 + 4x - 3)$ also yields two solutions. Please revise.

The screenshot shows a YouTube channel page for 'algebrametinzicht'. The channel has 0 subscribers and 431 video views. The page displays a list of videos, including 'Resultaten bekijken in de DWO', 'uitwerking 1 7', and 'uitwerking 2 2'. The channel description and upload history are also visible.

← Screencast clips

Feedback: worked examples and hints

√ □ □² □ (0) meer tip stap losop abc ↓ ↑

$(2x^2 - 3x - 2) \cdot (7x - 3) = (2x^2 - 3x - 2) \cdot (3x + 12)$

$2x^2 - 3x - 2 = 0$ of $7x - 3 = 3x + 12$

✓ □

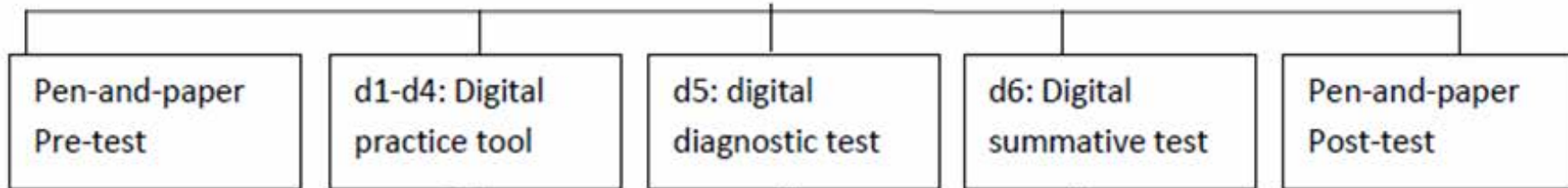
A*B=A*C geeft A=0 of B=C

Je bent goed aan het herschrijven. ×

A*B=A*C geeft A=0 of B=C

Fading

“too much of a dependency on feedback needs to be avoided”



Digital practice tool: every step is evaluated and appropriate feedback is presented.

$$(-4x + 3) \cdot (4x - 4) = (-4x + 3) \cdot -5$$

$$(4x - 4 + 5)(-4x + 3) = 0$$

$$(4x + 1)(3 - 4x) = 0$$

$$x = -\frac{1}{4} \text{ of } x = \frac{3}{4}$$

De vergelijking is correct opgelost.

Digital diagnostic test with self-assessment: the student chooses when to evaluate all steps of the task.

$$(-x - 2) \cdot (-x + 1) = (-x - 2) \cdot 2$$

$$(-x + 1 - 2)(-x - 2) = 0$$

$$(-x - 1)(-x - 2) = 0$$

$$x = -1 \text{ of } x = -2$$

Kijk na

Digital summative test: the student makes the exercises but doesn't see whether answers are wrong or right. Of course the teacher can access all the results. This corresponds with the situation in a pen-and-paper context.

More than sum of the parts

Let me say more on how these digital books could help mathematics education

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Authorable

Text: Title: New editor version | 800 px | 102

The need for another measure of spread

You can use this calculator:

First calculate the mean:

- Multiply the frequency f with x and fill in the third column.
- Calculate the sum of the frequencies $\sum f$ and the sum of $\sum f \cdot x$. Fill in at the bottom of columns two and three.
- Calculate the mean by dividing $\sum f \cdot x$ by $\sum x$. The mean is .
- Calculate the deviations from the mean $|x - \bar{x}|$ in the fourth column.

are in the t

- Calculate the squares of column four $|x - \bar{x}|^2$ and put them in the fifth column.
- Calculate $f|x - \bar{x}|^2$ and put in column six.

e in the
 e t

x	f	$f \cdot x$	$ x - \bar{x} $	$ x - \bar{x} ^2$	$f x - \bar{x} ^2$
#a#	#b#
#a + 1#	#c#
#a + 2#	#d#
#a + 3#	#f#
#a + 4#	#g#
#a + 5#	#h#
#a + 6#	#k#
#a + 7#	#l#
#a + 8#	#m#
#a + 9#	#n#
#a + 10#	#p#
	$\sum f =$	$\sum f \cdot x =$		$\sum x - \bar{x} ^2 =$	
	<input type="text"/>	<input type="text"/>		<input type="text"/>	

Variables for random parameters:

a=16

opdrachten Practice

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

More than sum of the parts

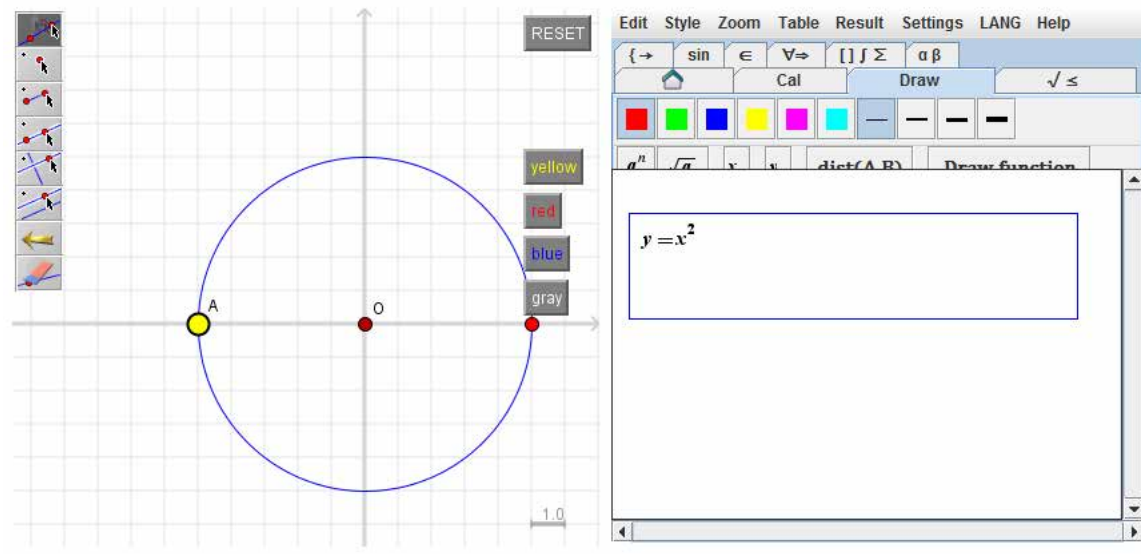
Let me say more on how these digital books could help mathematics education

1. Widgets
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Remains a challenge

- Between the ‘management system’ and individual widgets (e.g. SCORM)
- Between widgets (e.g. multiple representations)

1. First, you can consider the situation in the workspace in which $r = 5\text{cm}$. Use EpsilonWriter to suggest formulas that you can verify by using the “Draw function” button. However, you can only verify formulas in the form $y = f(x)$.



The image shows a geometry workspace on the left and the EpsilonWriter interface on the right. The workspace features a coordinate grid with a blue circle centered at the origin O . A yellow point A is located on the negative x-axis, and a red point is on the positive x-axis. A vertical line segment is drawn from the origin to the top of the circle. The EpsilonWriter interface has a menu bar (Edit, Style, Zoom, Table, Result, Settings, LANG, Help) and a toolbar with various mathematical symbols and functions. A text input field contains the formula $y = x^2$.

Here, Cinderella (left) can ‘talk’ with Epsilonwriter (right).

HTML5 player

No SIM 18:13 58%

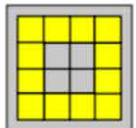
DWO Player Project

ws.fisme.science.uu.nl/dwo/tablet/#TreeModulePlace:31747

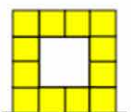
Nabouwen met drie aanzicht...

Opdracht 1
Maak het blokkenbouwsel waarvan je hieronder de drie aanzichten ziet. Als het lukt, dan krijg je 5 punten. Gebruik zo weinig mogelijk blokjes om de hoogste score van 10 punten te krijgen.

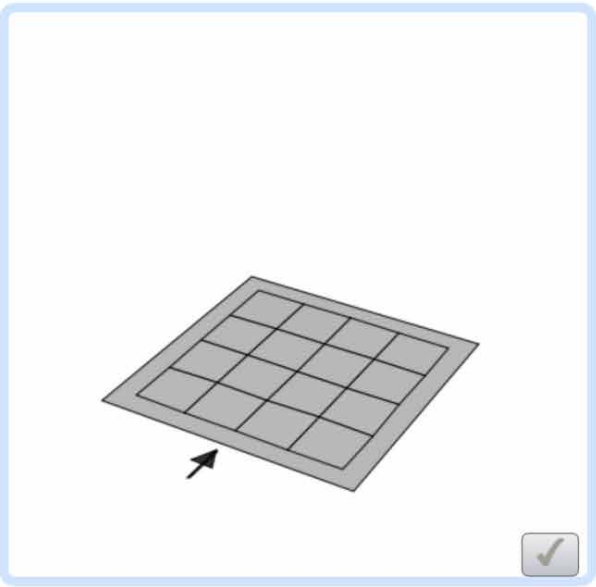
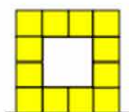
boven



voor



rechts



1 2 3 4 5 6 7 8 9 10

We combined them in the enGasia project

1. Compare geometry education in England, Japan and Hong Kong → some shown now.
2. two digital resources (electronic books) will be designed. They are then implemented in classrooms in those countries.
3. The methodology will include a more qualitative approach based on lesson observations and a quasi-experimental element.



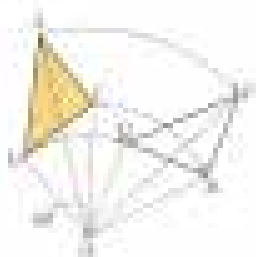
*** Rotation ***

When a shape is turned about a point, it is rotated about a central point O . The movement of that shape is called a rotation.

The central point O is called the center of rotation.

Q 2 Rotation

In the figure on the right, $\triangle PQR$ is rotated 90° clockwise from $\triangle ABC$ around the center of rotation O .



Q 4 In **Q 2**, what relationship do you observe between the lengths OA and OP , the segments that join the corresponding points A and P with the center of rotation O ?

Write an equation for the following about rotation:

All pairs of corresponding points are an equal distance from the center of rotation, and the angle of rotation equals the angle each pair to the center of rotation are observed.



Q 5 Construct the image that is rotated 90° from $\triangle ABC$ around the center of rotation O in **Q 2**.

Construction
 1. Draw line segments
 2. Measure their lengths

When the rotation 90° is called point symmetry rotation.

In every point O , a pair of corresponding points and the center of rotation lie about the same line.

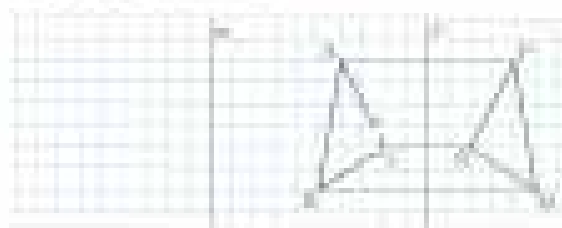


*** Reflection ***

When a shape is flipped over a straight line, the F image is shown. The movement of that shape is called reflection. The central line is called the mirror of reflection.

Q 1 Reflection

In the figure below, $\triangle PQR$ is reflected from $\triangle ABC$ and the axis of reflection is



Q 2 In **Q 1**, what relationship do you observe between AB , CD , and CE (lines connecting corresponding points and the axis of reflection F)?

Write an equation for the following about reflection:

1. Different shapes have the same size and the same rotation.
2. The segments joining corresponding points to the axis of reflection are perpendicular to F and their lengths from F are equal halves.



Q 3 Construct the image that is reflected across the axis of reflection in **Q 1**.

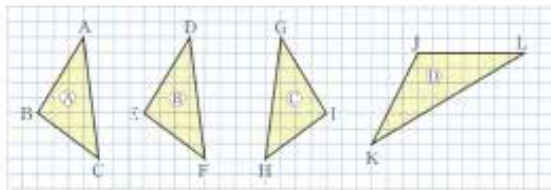
The design process

- Multiple widgets
- Some work 'in the backend' (e.g. feedback)

3 Congruent triangles

Learn how to determine when two triangles are congruent.

Which of the triangles below would line up exactly if we laid it on top of $\triangle ABC$? State which sides would match up.



Try flipping them too!

In above, triangles $\triangle A$ and $\triangle B$ line up exactly, so they are congruent. $\triangle A$ and $\triangle C$ are also congruent.

The vertices, sides, and angles that line up in congruent figures are called *corresponding* vertices, sides, and angles.

We can say the following about congruent figures.

Properties of congruent figures

- 1 The corresponding segments of congruent figures are equal in length.
- 2 The corresponding angles of congruent figures are equal in measure.

We can express the fact that quadrilateral $AECD$ is congruent to

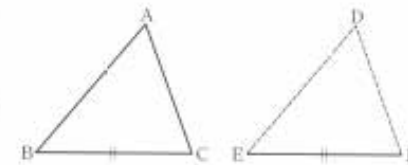


◆◆◆ Conditions for congruent triangles ◆◆◆

Extend What should we do?

How can we draw $\triangle DEF$, which is congruent to $\triangle ABC$?

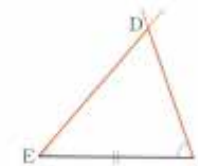
We started by drawing side EF , which is the same length as side BC . How can we determine where vertex D is?



As the figure on the right shows, we can determine a unique $\triangle DEF$ if we position point D so that in addition to $EF = BC$,

$$\angle E = \angle B, \angle F = \angle C.$$

If we line up side EF with side BC of $\triangle ABC$, then vertex D lines up with A . This tells us that $\triangle DEF$ is congruent to $\triangle ABC$.

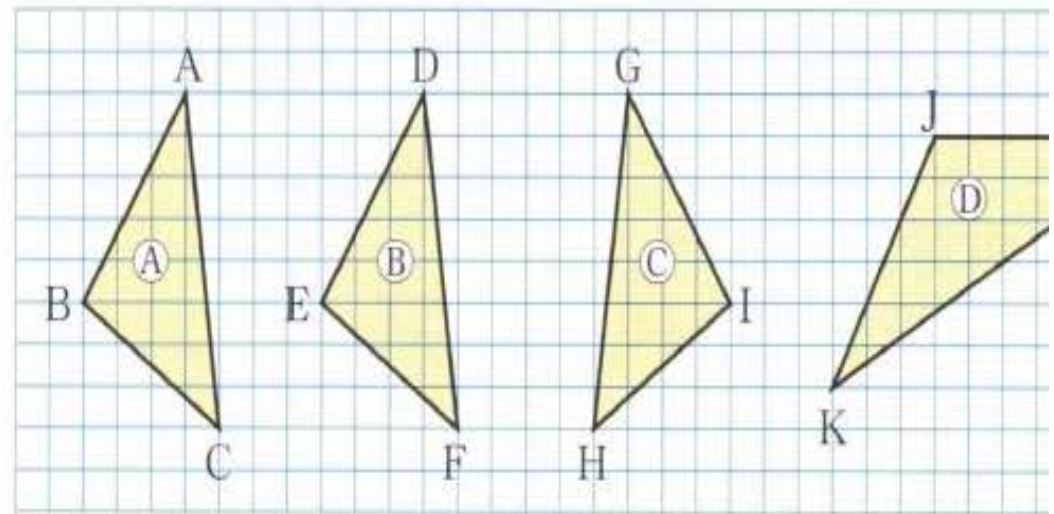


Q 2 Draw $\triangle DEF$ in by determining point D so that in addition to $EF = BC$, $\angle E = \angle B$ and $DE = AB$.



This could be a geogebra widget but perhaps not necessary. More important is feedback.

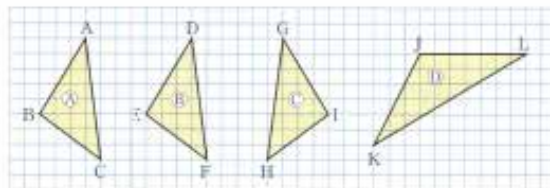
Which of the triangles below would line up exactly if we laid it on top of $\triangle ABC$? State which



3 Congruent triangles

Learn how to determine when two triangles are congruent.

Which of the triangles below would line up exactly if we laid it on top of $\triangle ABC$? State which sides would match up.



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Properties of congruent figures

- 1 The corresponding segments of congruent figures are equal in length.
- 2 The corresponding angles of congruent figures are equal

In the diagram above triangles A and B line up exactly, so they are congruent. A and C are also congruent. Sides and angles that line up in congruent figures are called **corresponding** vertices, sides and angles. What

Side AB aligns with sides and

Side BC aligns with sides and

Side CA aligns with sides and

You probably thought that the side on the left would be the correct side. However, have a look at the position of the angles.

Page: 1 2 3 4 5 6 7 8

Score: 2

Total: 2

Flowchart (Prof. Miyazaki)

Let's try flowchart thinking

Drag & Drop

History

CHECK!

Select the proper one.

$OA = OB$

$\triangle ADO \equiv \triangle BCO$

CONTENTS

Lesson1

Lesson2

Lesson3

http://www.sun-first.jp/fc_html5d/

Conclusions

- Technology sometimes works for mathematics education, and sometimes it doesn't
- Need to think about what you want to achieve
- I showed some examples: feedback, representations, storing student work, etc.
- The enGasia project tries to combine some of these in digital mathematics books
- Only a small part of the possibilities.....