UNIVERSITY OF Southampton
School of Education

Using technology for maths teaching and learning: Instructional design, digital books and automated feedback

Dr. Christian Bokhove

Shinshu University – 1 March 2017

Contents

- Some general comments about technology use
- Does technology work for maths education
  - Yes, but under certain conditions
  - Integrate in ‘digital books’
- Features of those digital books
- Examples of technology
  - I have added many, I will not cover all of them.

Who am I

- Dr. Christian Bokhove
- From 1998-2012 teacher maths, computer science, head of ICT secondary school Netherlands
- PhD ‘Use of ICT for acquiring, practicing and assessing algebraic expertise’ with Prof. Van Maanen and Prof. Drijvers
- Since 2012 Lecturer at University of Southampton
  - Maths education
  - Technology use
  - Large-scale assessment

INTRODUCTION

There is a lot to do with technology

On average, in the past 10 years there has been no appreciable improvement in student achievement in reading, mathematics or science in the countries that have invested heavily in information and communication technologies for education.

In Australia, New Zealand and the United Kingdom, **every 15-year-old has individual access to a computer at school.**

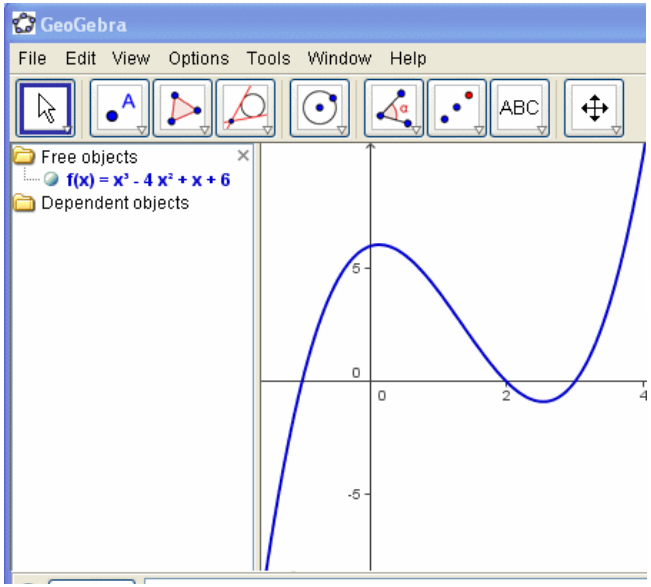
In Germany, Italy and Japan, there is **only one school computer available for every four 15-year-old students.**

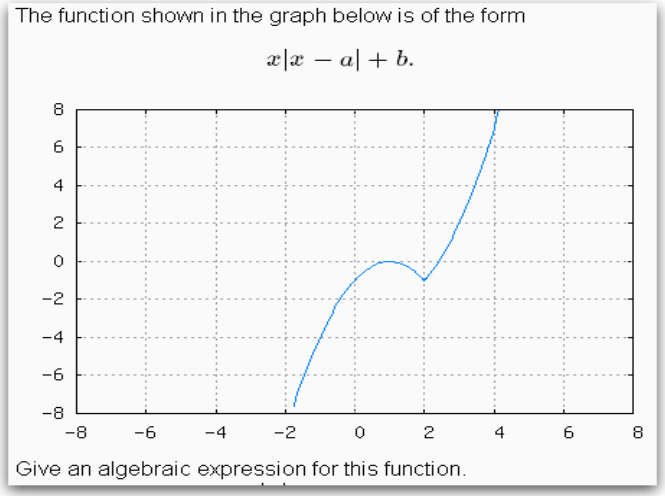
OECD

http://www.keepeek.com/Digital-Asset-Management/oecd/education/students-computers-and-learning_9789264239555-en#.V8wy2jV5-aU

In mathemetics education…

- 17th ICMI study “Mathematics Education and Technology-Rethinking the Terrain”
- 22nd ICMI study “Task Design In Mathematics Education”
- …





http://timss2015.org/

TIMSS & PIRLS

IEA TIMSS & PIR International Study Ce Lynch School of Education, Boston

Home About TIMSS PIRLS Publications International Databases Contact News Search this report... Searc

TIMSS 2015 AND TIMSS ADVANCED 2015 INTERNATIONAL RESULTS

SAVED PAGES

MATHEMATICS SCIENCE ABOUT TIMSS 2015 DOWNLOAD CENTER SITEMAP TIMSS ADVANCED 201

PRESS RELEASE

Chestnut Hill, Mass. (11/29/2016) — Singapore, Hong Kong SAR, Korea, Chinese Taipei, and Japan continue outperforming all participating countries in mathematics at the fourth and eighth grades, maintaining a 20 year edge according to results released today from TIMSS, the longest running, large scale international assessment of mathematics and science education in the world.

Read more

Highlights from TIMSS 2015

Exhibit 9.6: Computer Activities During Mathematics Lessons

Reported by Teachers

Country	Computers Available for Students to Use in Mathematics Lessons			Percent of Students Whose Teachers Have Them Use Computers at Least Monthly			
	Percent of Students	Average Achievement					
	Yes	Yes	No	To Explore Mathematics Principles and Concepts	To Practice Skills and Procedures	To Look Up Ideas and Information	To Process and Analyze Data
Sweden	65 (3.6)	499 (4.0)	502 (4.0)	25 (3.7)	38 (4.0)	32 (4.2)	26 (3.9)
Australia	62 (3.4)	512 (3.5)	506 (5.4)	51 (3.5)	52 (3.6)	48 (3.6)	44 (3.2)
Kazakhstan	53 (3.9)	531 (7.6)	525 (7.4)	45 (4.5)	50 (4.1)	51 (4.2)	45 (4.5)
Canada	50 (3.3)	528 (3.7)	533 (3.2)	35 (2.8)	36 (3.1)	33 (3.0)	31 (3.1)
Chile	49 (4.6)	423 (5.5)	437 (5.8)	29 (4.3)	36 (4.4)	32 (4.5)	36 (4.3)
Egypt	48 (3.9)	395 (6.1)	390 (5.8)	35 (4.0)	42 (4.1)	45 (4.0)	32 (3.7)
Russian Federation	47 (3.5)	535 (5.1)	540 (6.4)	36 (3.5)	41 (3.6)	42 (3.2)	34 (3.5)
New Zealand	47 (3.5)	501 (4.8)	488 (5.7)	36 (3.3)	35 (3.3)	35 (3.3)	33 (3.5)
United Arab Emirates	44 (2.2)	481 (4.5)	456 (3.8)	38 (2.0)	40 (2.1)	40 (2.2)	37 (2.3)
Japan	43 (3.7)	585 (4.1)	588 (3.4)	3 (1.0)	6 (1.8)	4 (1.3)	5 (1.5)
Italy	43 (3.7)	493 (4.3)	495 (4.1)	28 (3.2)	29 (3.3)	31 (3.5)	26 (2.9)
Norway (9)	40 (3.9)	513 (3.5)	513 (3.2)	27 (3.9)	35 (4.1)	27 (4.0)	29 (3.8)
Jordan	39 (3.3)	394 (6.5)	378 (4.0)	29 (3.4)	28 (3.4)	32 (3.5)	25 (3.3)
Thailand	39 (4.5)	442 (8.5)	425 (6.1)	25 (4.0)	26 (4.2)	28 (4.3)	23 (4.1)
United States	39 (2.9)	519 (5.0)	518 (4.3)	r 27 (2.8)	r 31 (2.9)	r 29 (2.8)	r 26 (2.8)
Korea, Rep. of	39 (3.6)	604 (4.3)	607 (3.6)	25 (3.3)	22 (3.1)	24 (3.2)	19 (2.6)
Lithuania	38 (4.0)	508 (4.9)	512 (4.5)	21 (3.7)	24 (3.4)	29 (3.8)	23 (3.5)
Georgia	38 (3.6)	453 (6.6)	452 (4.5)	33 (3.8)	31 (3.5)	34 (3.8)	33 (3.7)
Qatar	36 (2.6)	422 (6.6)	445 (4.3)	31 (2.3)	33 (2.5)	30 (2.7)	26 (2.9)
Singapore	35 (2.5)	617 (6.0)	621 (4.1)	27 (2.2)	27 (2.3)	23 (2.0)	19 (2.0)
Hungary	30 (3.8)	509 (8.0)	516 (4.6)	20 (3.3)	27 (3.6)	22 (3.2)	18 (3.0)
Bahrain	30 (2.8)	458 (3.8)	452 (2.2)	23 (2.4)	23 (2.7)	24 (2.8)	16 (1.9)
England	29 (4.1)	511 (9.7)	520 (6.0)	17 (3.6)	23 (3.7)	17 (3.3)	13 (2.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2015

Exhibit 9.7: Student Use of Internet for Schoolwork

Reported by Students

Country	Percent of Students Who Use the Internet to Do the Following Tasks				
	Access the Textbook or Other Course Materials	Access Assignments Posted Online by the Teacher	Collaborate with Classmates on Assignments or Projects	Communicate with the Teacher	Find Information, Articles, or Tutorials to Aid in Understanding Mathematics
Australia	55 (1.4)	66 (1.2)	63 (0.8)	46 (1.1)	57 (1.0)
Bahrain	56 (1.0)	43 (1.1)	77 (0.8)	41 (1.1)	58 (0.8)
Botswana (9)	46 (0.8)	37 (1.0)	58 (1.1)	36 (0.8)	54 (0.8)
Canada	45 (1.5)	58 (2.0)	76 (1.0)	32 (1.2)	56 (1.2)
Chile	62 (1.0)	37 (1.4)	79 (0.9)	25 (1.2)	60 (1.0)
Chinese Taipei	74 (0.9)	50 (1.1)	72 (1.0)	28 (1.0)	38 (0.8)
Egypt	57 (1.1)	34 (1.0)	58 (1.0)	56 (1.2)	64 (1.0)
England	54 (1.5)	71 (1.4)	53 (1.4)	33 (1.9)	66 (1.1)
Georgia	76 (1.3)	44 (1.5)	73 (1.3)	31 (1.4)	47 (1.2)
Hong Kong SAR	51 (1.3)	64 (1.9)	76 (1.3)	33 (1.2)	61 (1.1)
Hungary	40 (1.1)	58 (1.2)	76 (1.1)	26 (1.3)	41 (1.1)
Iran, Islamic Rep. of	60 (1.4)	40 (1.1)	56 (1.2)	31 (1.0)	52 (1.2)
Ireland	34 (1.2)	35 (2.6)	50 (1.2)	12 (1.2)	44 (1.0)
Israel	64 (1.2)	68 (1.4)	60 (1.2)	32 (1.3)	55 (0.9)
Italy	50 (1.1)	34 (2.1)	75 (1.1)	27 (1.5)	41 (1.0)
Japan	23 (0.8)	16 (0.9)	28 (1.0)	5 (0.5)	30 (0.8)
Jordan	65 (1.1)	42 (1.2)	70 (1.3)	49 (1.2)	61 (1.0)
Kazakhstan	65 (1.1)	39 (1.5)	76 (0.9)	24 (1.3)	66 (0.9)
Korea, Rep. of	51 (1.0)	43 (1.3)	69 (1.1)	13 (0.7)	45 (0.9)
Kuwait	x x	x x	x x	x x	x x

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2015

- So there are big differences between countries…
- It is not clear when technology contributes positively
- My research in the last years has focussed on finding out a bit more…
- I will give some examples of ways that seem positive.

In 2010 I looked at specific criteria for algebra software

Table 2 The top five of most important and bottom five of least important criteria

Rank	Description	Weight
1	The stability and performance of the tool	4.89
2	The tool is easy to use for a student (e.g. equation editor, short learning curve, interface)	4.85
3	The tool is able to display formulas correctly	4.81
4	The tool is mathematically sound and faithful to the underlying mathematical properties (e.g. conventional representations, sound operations)	4.74
5	The tool stores the answers given by a student	4.70
23	The tool has the ability to randomize algebra assignments	3.96
24	The cost of the tool	3.74
25	The tool makes use of standards (e.g. QTI, SCORM)	3.72
26	The tool enables the student to use a computer algebra system as a tool	3.63
27	The licensing of the tool (e.g. open, proprietary)	3.41

Bokhove, C., &Drijvers, P. (2010). Digital tools for algebra education: criteria and evaluation. *International Journal of Computers for Mathematical Learning, 15*(1), 45-62.

What should technology look like

This has developed over the years:

- From distinct, separate tools to integrated
- From offline towards online
- From not storing data to storing data

It became more and more clear that it made sense to make an INTEGRATED design: digital books.

INTEGRATED DESIGN: DIGITAL BOOKS

International Conference on Mathematics Textbook Research and Development 2014. Strand on e-Textbooks and technology. From Prof. Yerushalmy's talk

e-Textbook
Necessary Dimensions of Design

- **Interactive** engagements for students and teachers
  interactions amongst learners and between learner and the textbook

- Flexible **Integration**
  integration of 'adds-on' traditionally not part of textbooks
  input/output from and to external systems

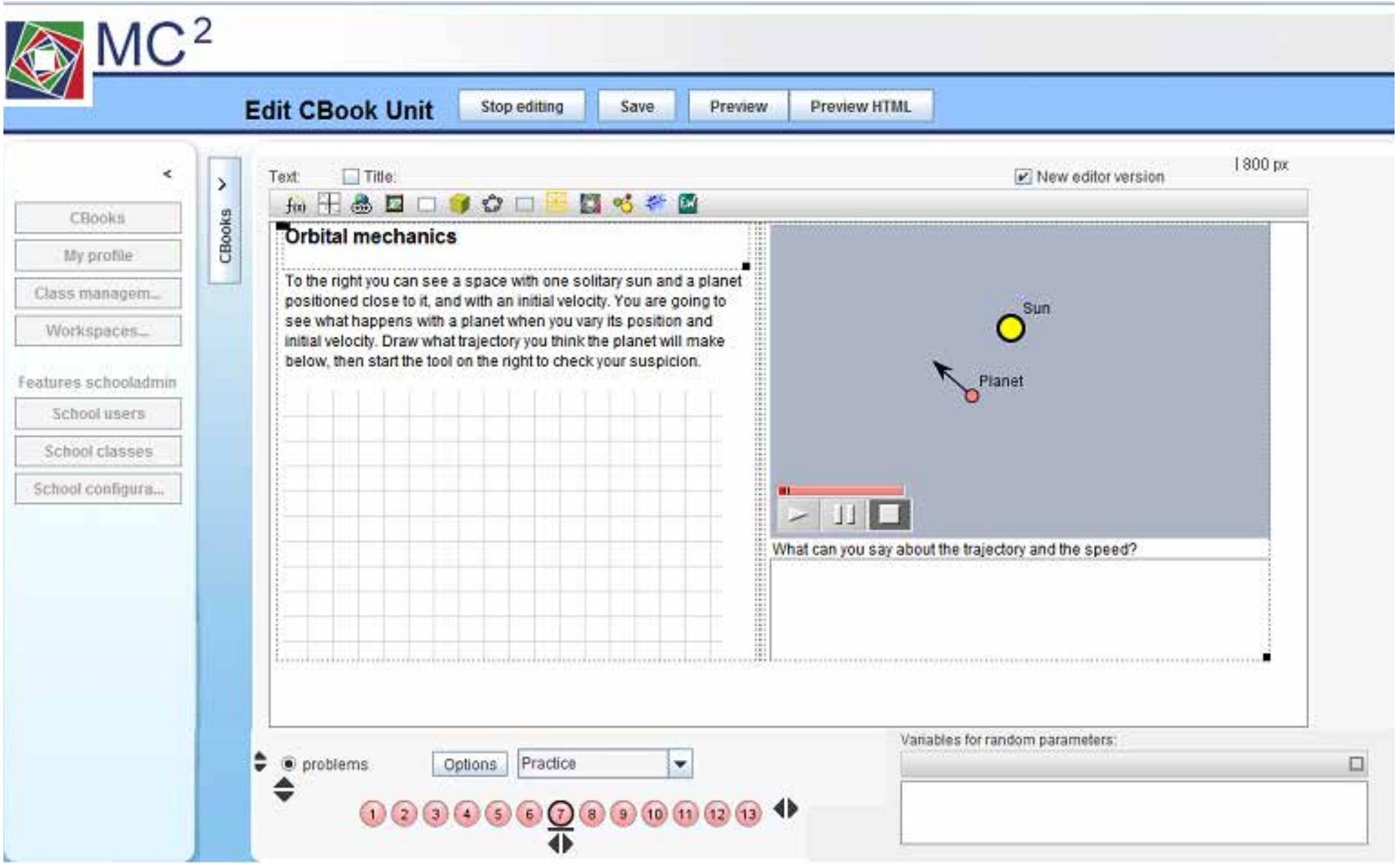
- **Evolving** continuously by its authors/users
  interactions amongst authors and users

Examples

- Apple iBooks
- Great looks
- Limited interaction
- Limited student management options

Towards digital textbooks

- Digital textbook: theory, examples, explanations
- Interactive content (in widgets)
- Interactive quizzes (formative assessment, feedback)
- Integrated workbook


MC2
Edit CBook Unit
Stop editing
Save
Preview
Preview HTML
CBooks
My profile
Class managem...
Workspaces...
Features schooladmin
School users
School classes
School configura...
CBooks
Text:
Title:
New editor version
800 px
Orbital mechanics
To the right you can see a space with one solitary sun and a planet positioned close to it, and with an initial velocity. You are going to see what happens with a planet when you vary its position and initial velocity. Draw what trajectory you think the planet will make below, then start the tool on the right to check your suspicion.
Sun
Planet
What can you say about the trajectory and the speed?
problems
Options
Practice
1 2 3 4 5 6 7 8 9 10 11 12 13
Variables for random parameters:

More than sum of the parts

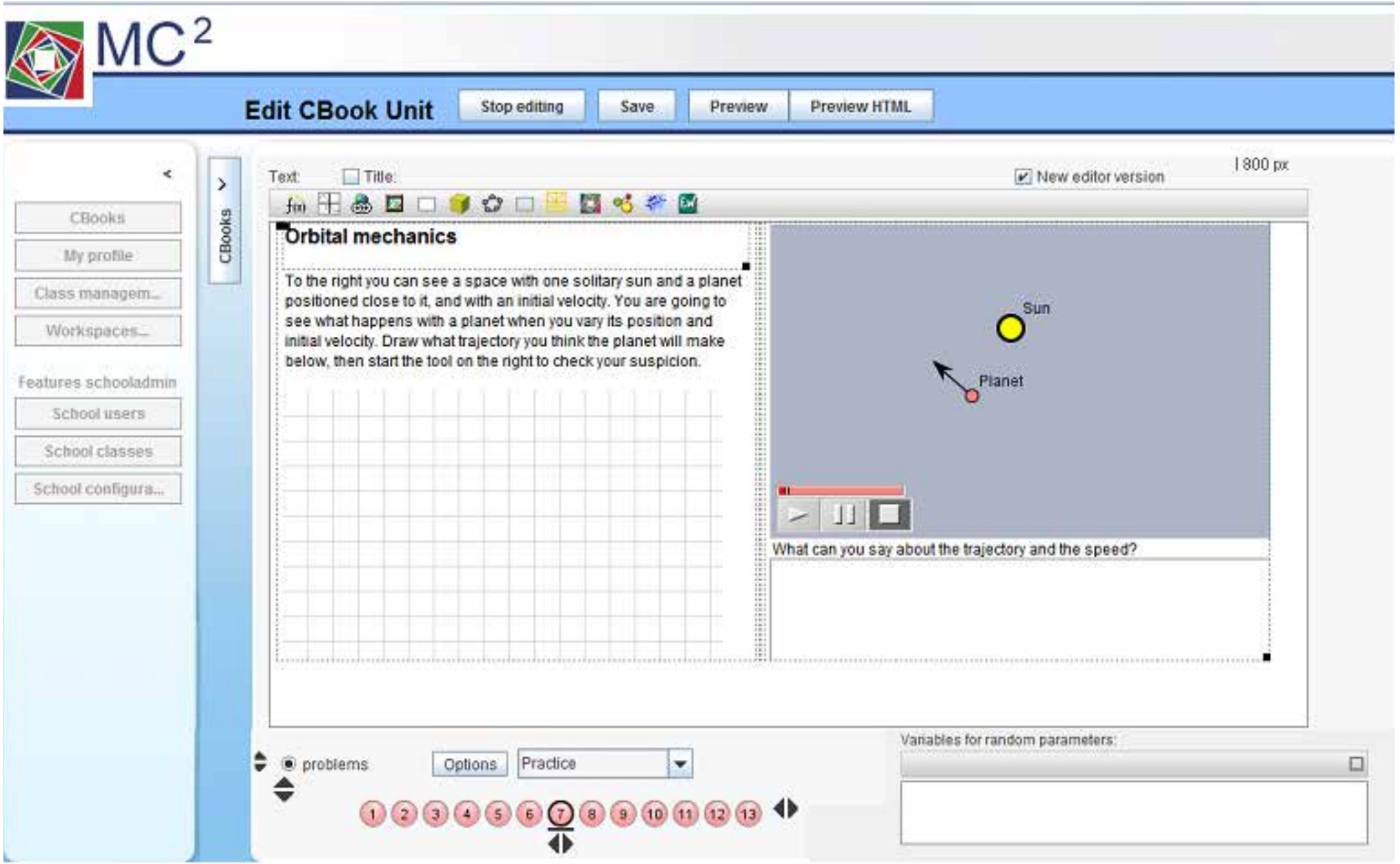
Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. Storing results
4. Instructional design
5. Authorability
6. Interoperability and standards

More than sum of the parts

Let me say more on how these digital books could help mathematics education

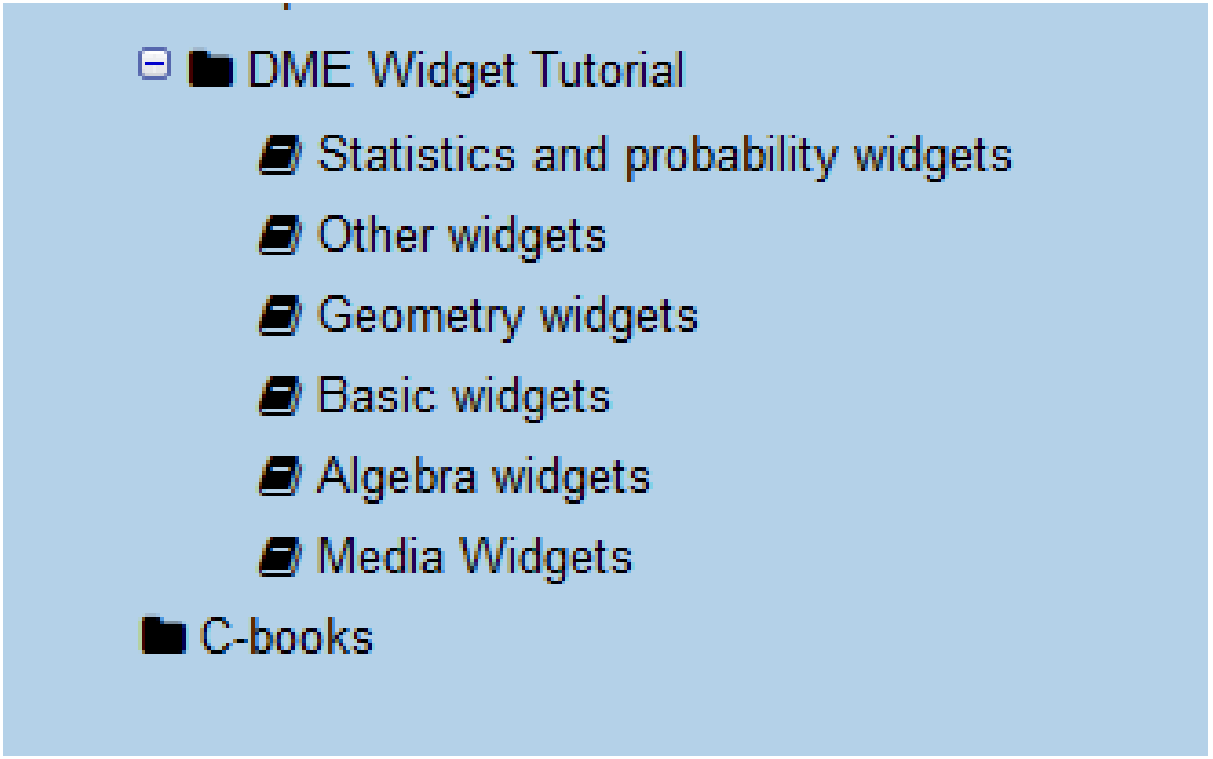
1. **Widgets**
2. Interaction (feedback)
3. Storing results
4. Instructional design
5. Authorability
6. Interoperability and standards


MC2
Edit CBook Unit
Stop editing
Save
Preview
Preview HTML
CBooks
My profile
Class managem...
Workspaces...
Features schooladmin
School users
School classes
School configura...
CBooks
Text:
Title:
New editor version
800 px
Orbital mechanics
To the right you can see a space with one solitary sun and a planet positioned close to it, and with an initial velocity. You are going to see what happens with a planet when you vary its position and initial velocity. Draw what trajectory you think the planet will make below, then start the tool on the right to check your suspicion.
Sun
Planet
What can you say about the trajectory and the speed?
problems
Options
Practice
1 2 3 4 5 6 7 8 9 10 11 12 13
Variables for random parameters:

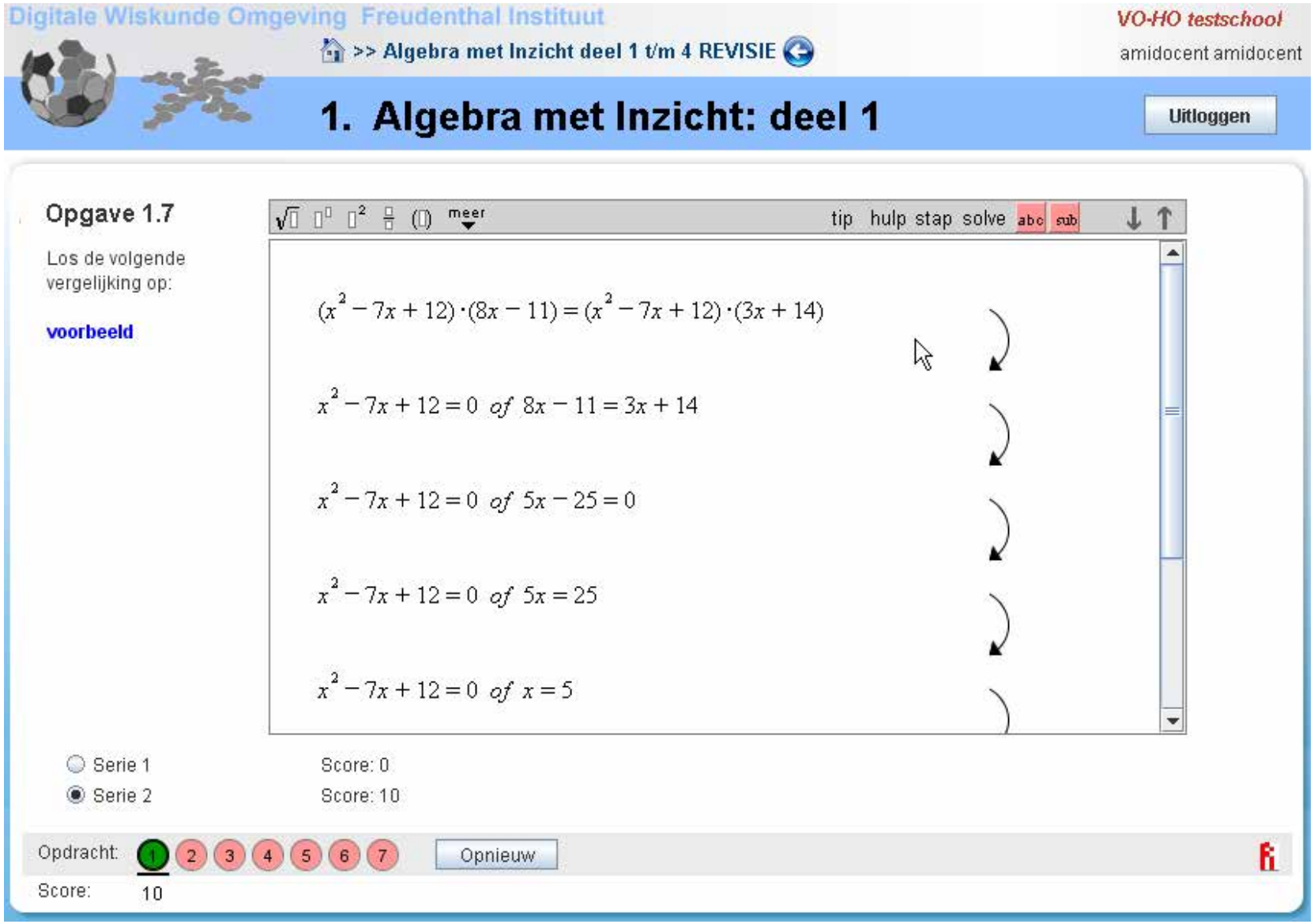
- Purpose of technology
  - e.g. tension learning and doing (e.g. Ainley & Pratt, 2002)
  - Use to learn, learn to use
- Openness
  - Beeson (1998), open and closed
  - Bliss & Ogborn (1998), exploratory and expressive
  - Buchberger (1990), black box, white box, glass box
- Interactivity

DEMO WIDGETS

- Browse some widgets
- http://mc2dme.appspot.com/



Widget example: equations



Basic Widgets

General Properties

Score: Points:

- [x] Equivalent 0
- [ ] Form
- [x] Answer needed 10
- [ ] Exact

Randomized variables

$\#a\# \cdot \#b\# \cdot x^{\#p\#}$

$\#a \cdot b\# \cdot x^{\#p\#}$

$\#a \cdot b\# \cdot \#x^{p}\#$

$\#a \cdot b \cdot x^{p}\#$

Stepwise Formula Answer Box

more

$3x^2 + 2x + x^2 - x + 3 + 4x - 2 =$

$3x^2 + x^2 + 2x - x + 4x + 3 - 2 =$

$4x^2 + 5x + 1$

✓

Stepwise Equation Answer Box

more

$\frac{1}{4}x + 7 = 5x - 2$ ×4

$3x + 28 = 20x - 8$ - 3x

$28 = 17x - 8$ + 8

$36 = 17x$

Small Formula Answer Box

$5 \cdot 3 =$ [ ]

Small Equation Answer Box

Solve the equation: $x^2 - 3x + 2 = 0$.

[ ]

Text Answer Box

f(x)

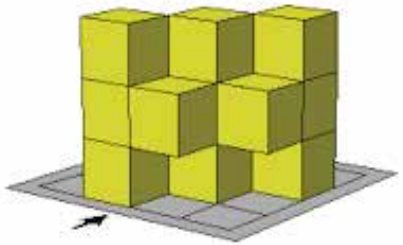
Check Text Answer Box

Which day comes after Sunday?

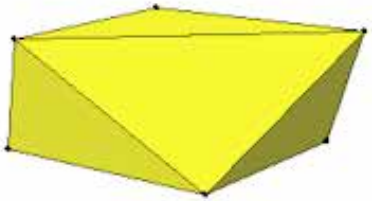
[ ]

Geometry Widgets

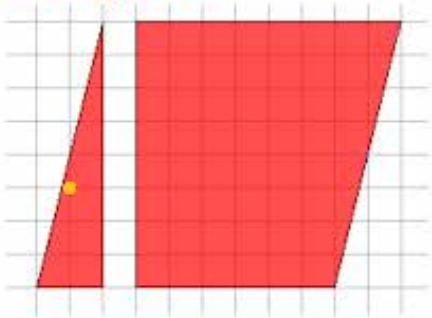
Building Blocks



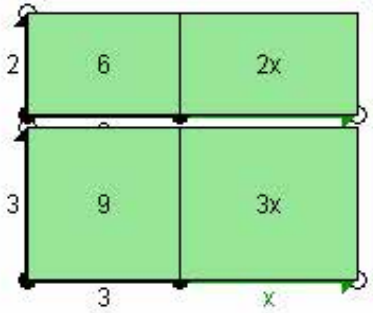
3D Construction tool



Cutting Areas

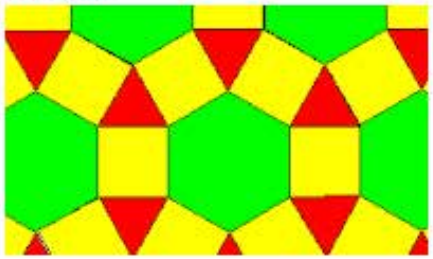


2D Geometrical Algebra



Drawing in Space

Mozaik

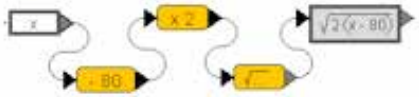


Tilings

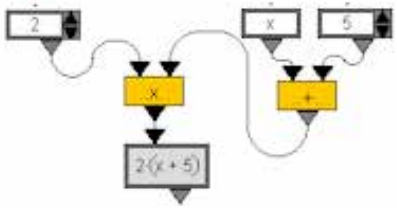
Area Algebra

Algebra Widgets

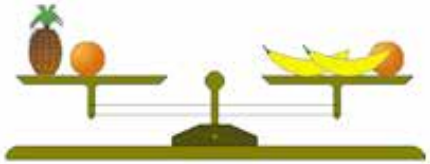
Algebra Arrows



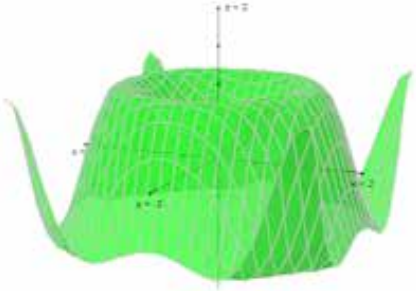
Algebra Trees



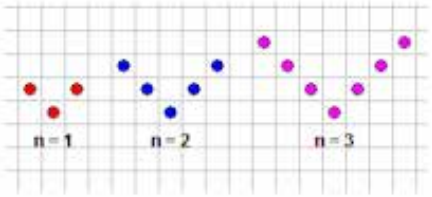
(Fruit) Balance



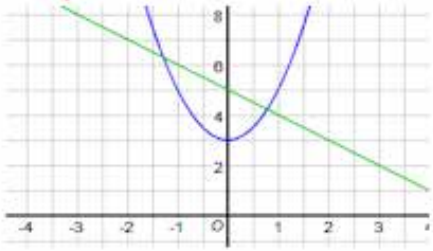
3D Graphing



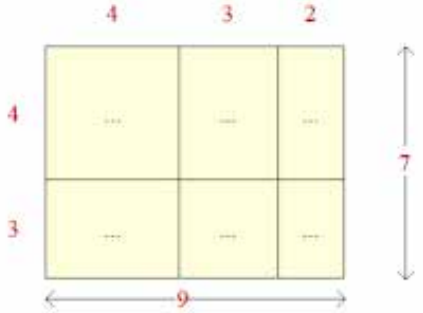
Patterns with Dots



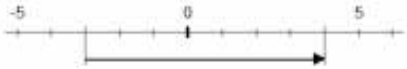
Graph Tool



Area Algebra

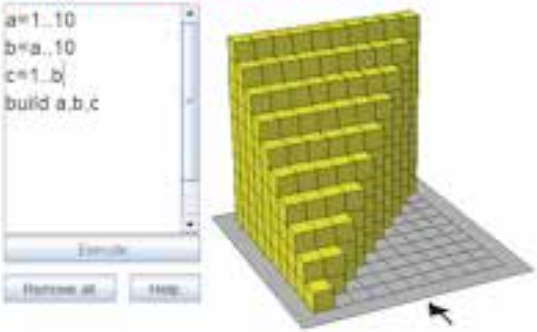


Number Line

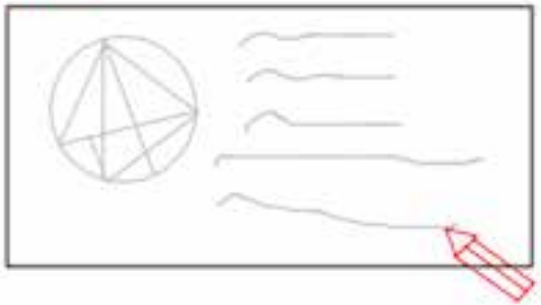


Other Widgets

Building Programs



Note

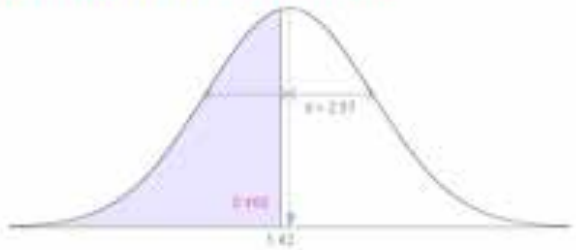


C-Book Units

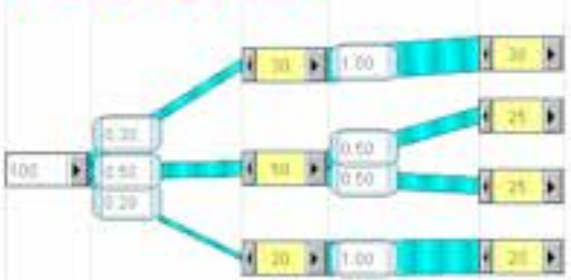
1. Building Programs
2. Note

Statistics and Probability Widgets

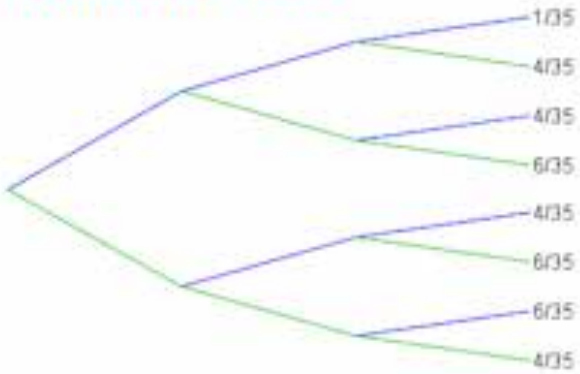
Normal Distribution



Flow Diagrams



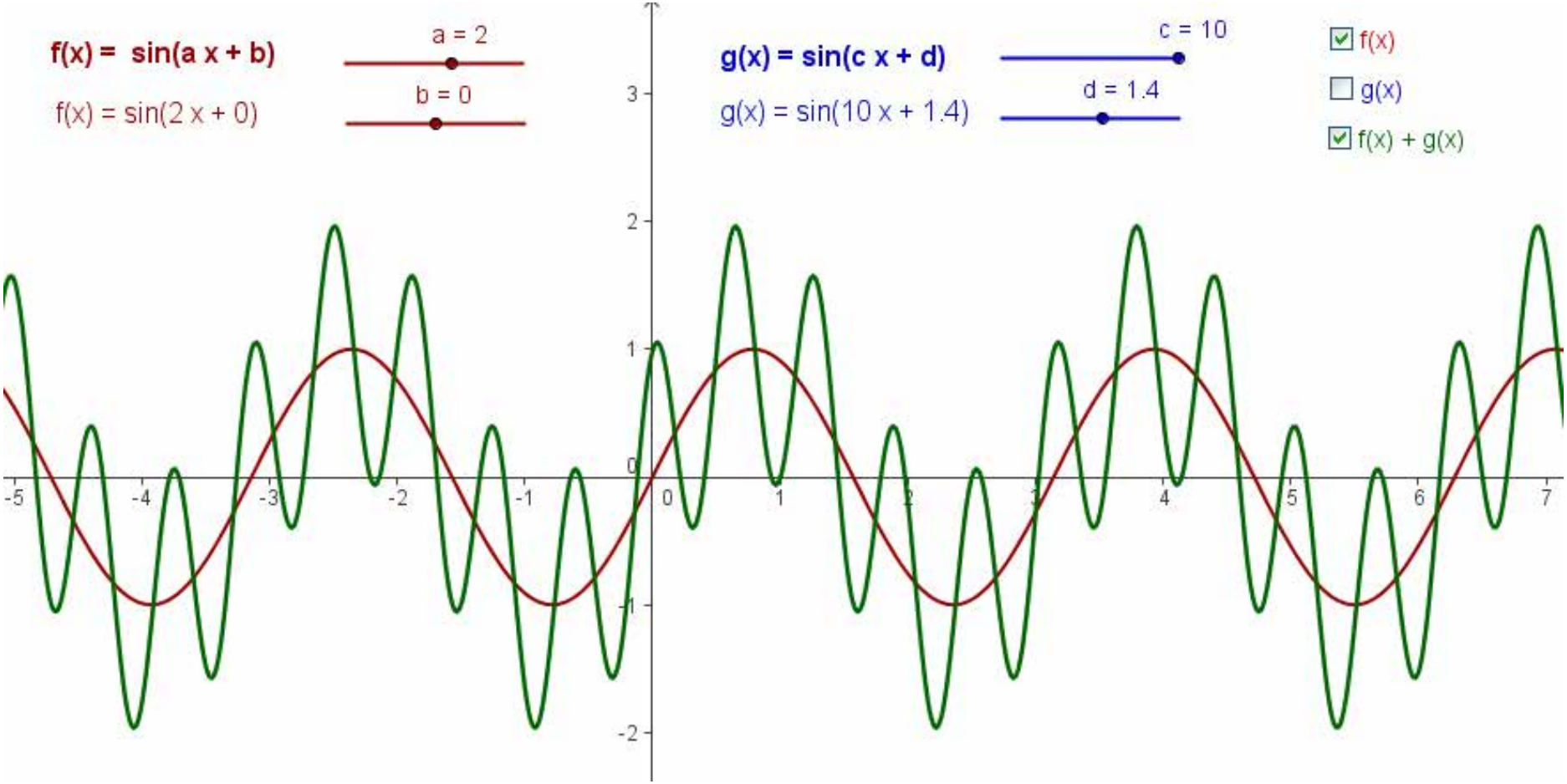
Probability Trees



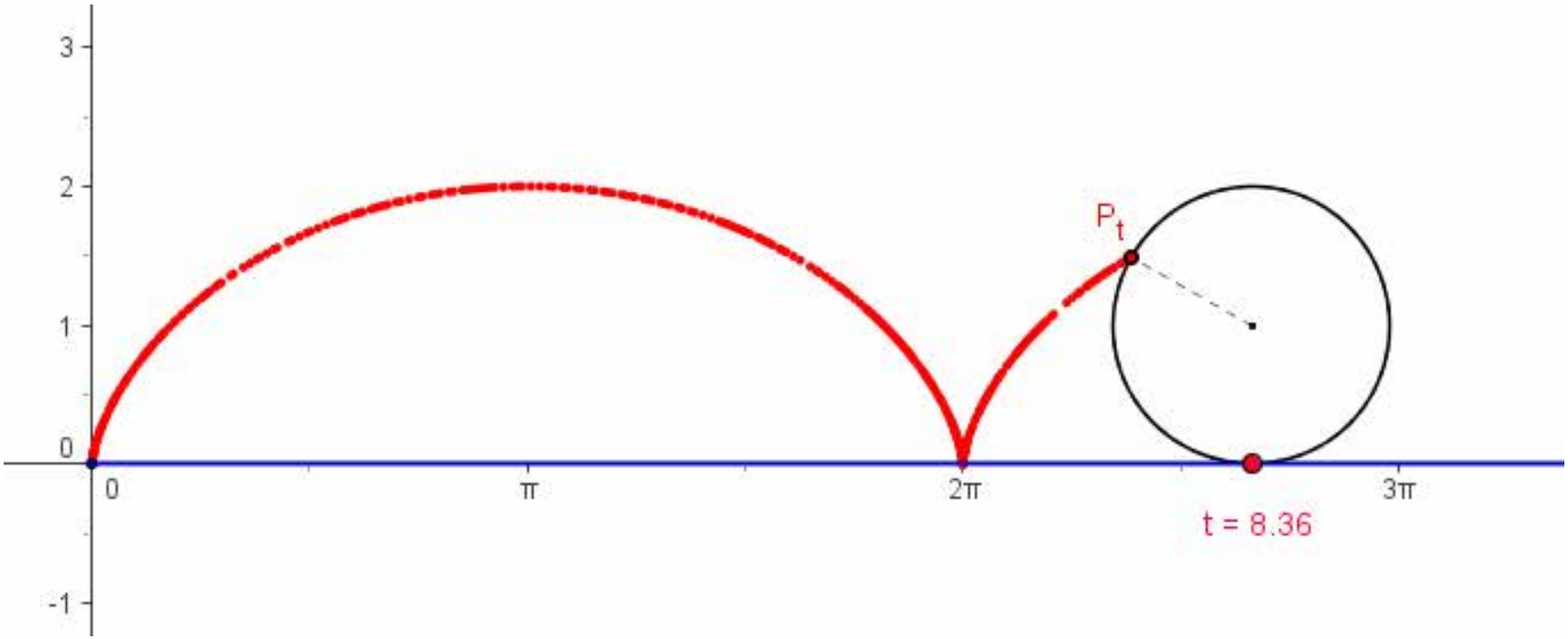
What could ‘open’ widgets add?

- **Visualizing**
  students can “see” abstract concepts
- **Representations**
  students can make connections
- **Experiments**
  students can discover mathematics

Example: geogebra
Visualizing Parameters



Example: geogebra
Visualizing Movement



Multiple Representations

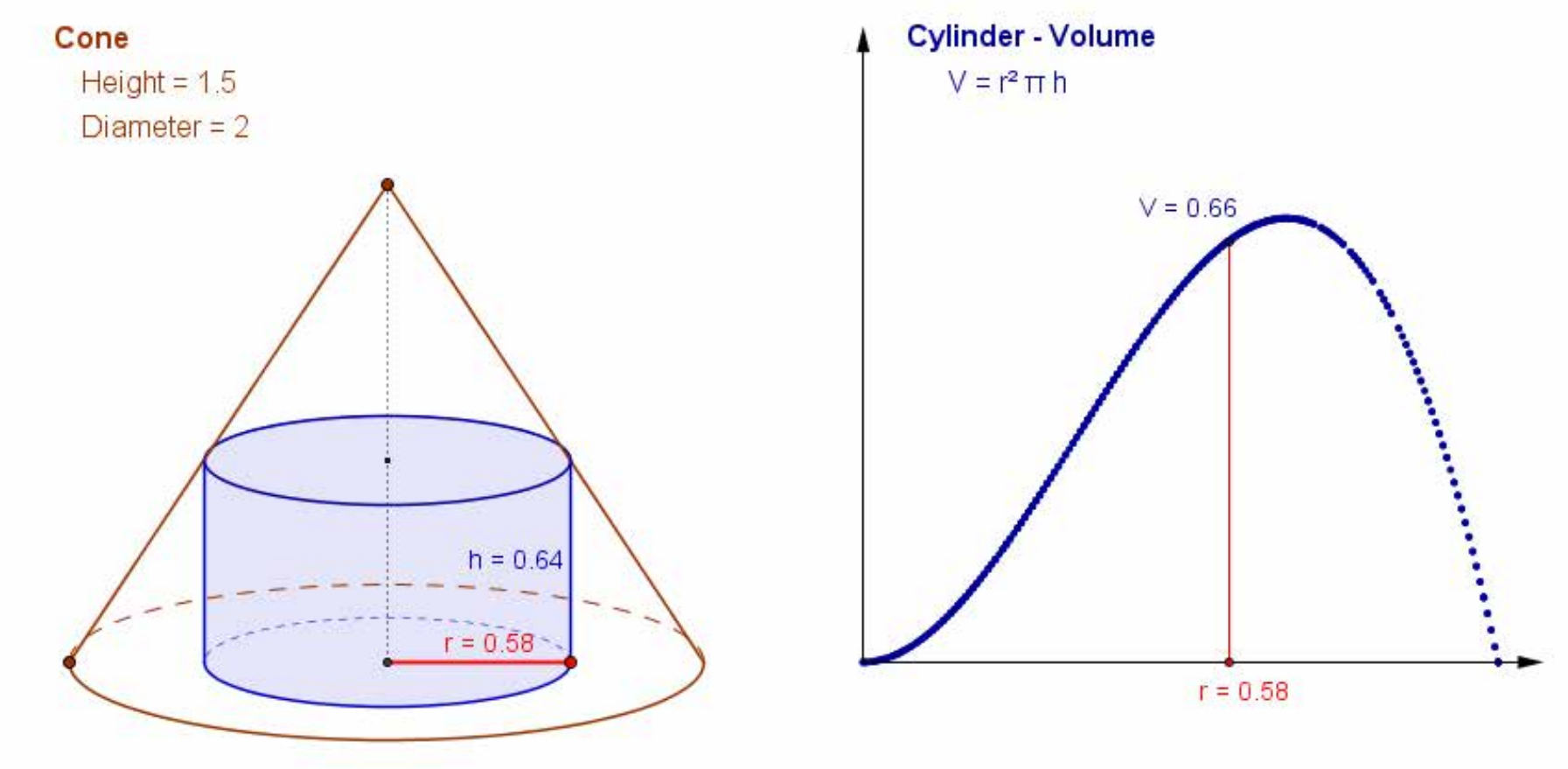
- GeoGebra: Bidirectional Connection

  Symbolic ⇔ Graphics

- Examples
  - Coordinates ⇔ Point in Coordinate System
  - Circle‘s Equation ⇔

    Circle drawn in Coordinate System
  - Function‘s Equation ⇔ Graph of Function

Example: geogebra
Representations: Sketch & Graph



More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. **Interaction (feedback)**
3. Storing results
4. Instructional design
5. Authorability
6. Interoperability and standards

Feedback

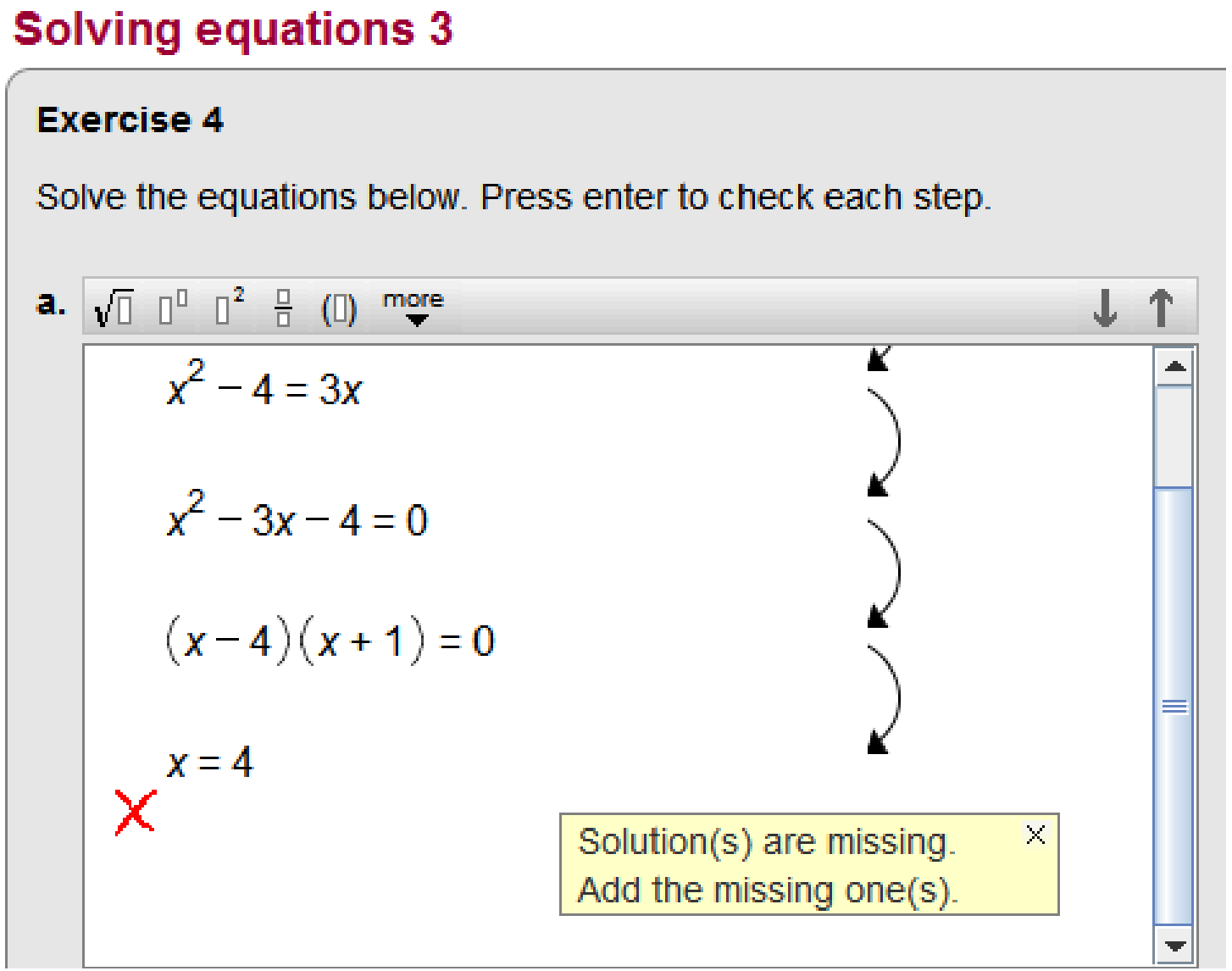
- Essential part formative assessment (Black & William, 1998)
- But depends on feedback type(s) e.g. task, process, self-regulation, self (Hattie & Timperley, 2007)
  - Task e.g. ‘that is the wrong answer’
  - Process e.g. ‘have another look at what you did’
  - Self-regulation e.g. ‘review your answers and decide what topic you want to revise’
  - Self e.g. ‘well done!’
- Issues
  - Positive/negative feedback
  - Timing

Feedback in computer systems

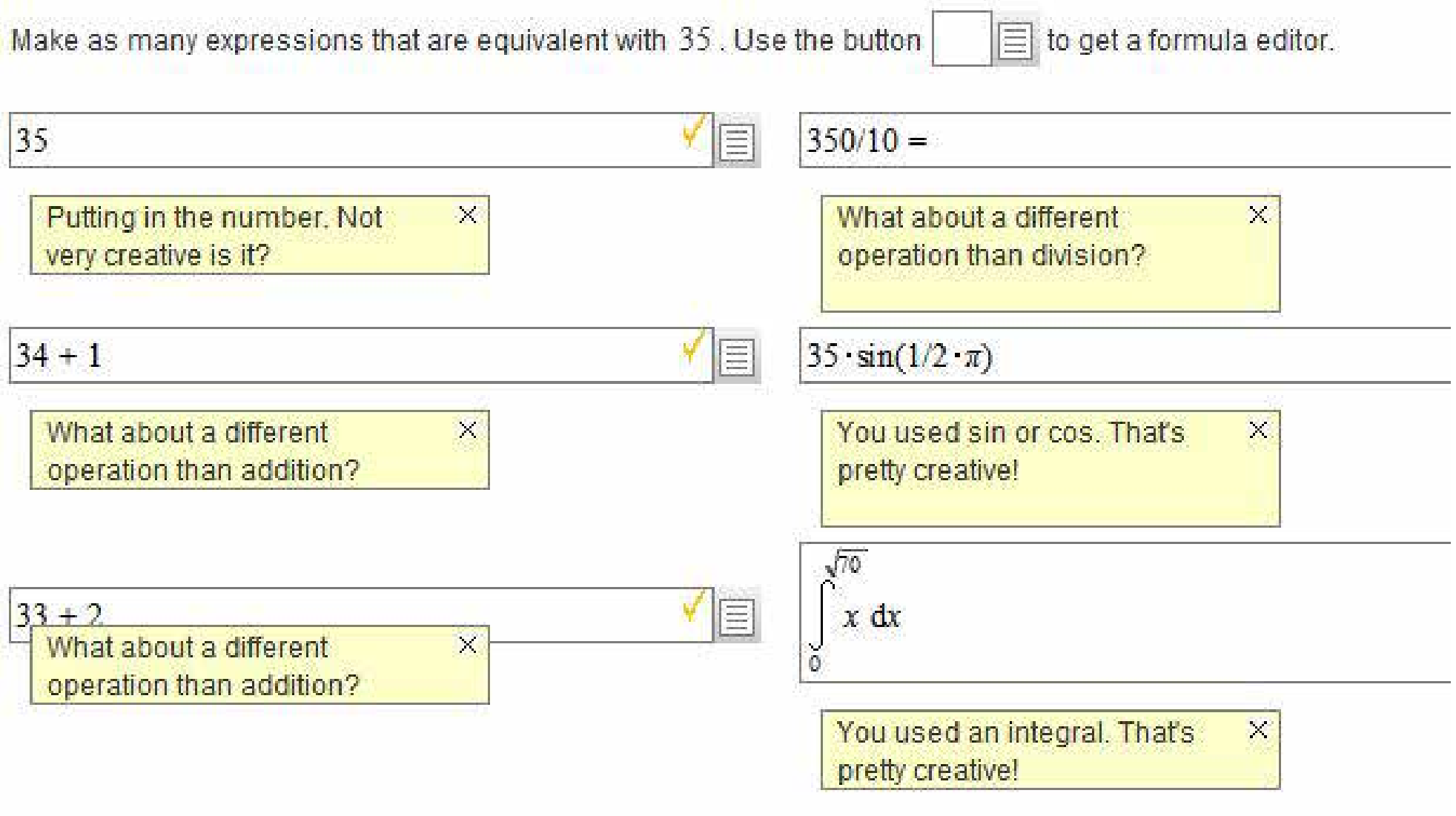
Review Vander Kleij et al. (2015)

- Types
  - Elaborated feedback 0.49
  - Providing the correct answer 0.32
  - Correctness of answer 0.05
- Higher order learning even more, as well as for mathematics compared to social sciences

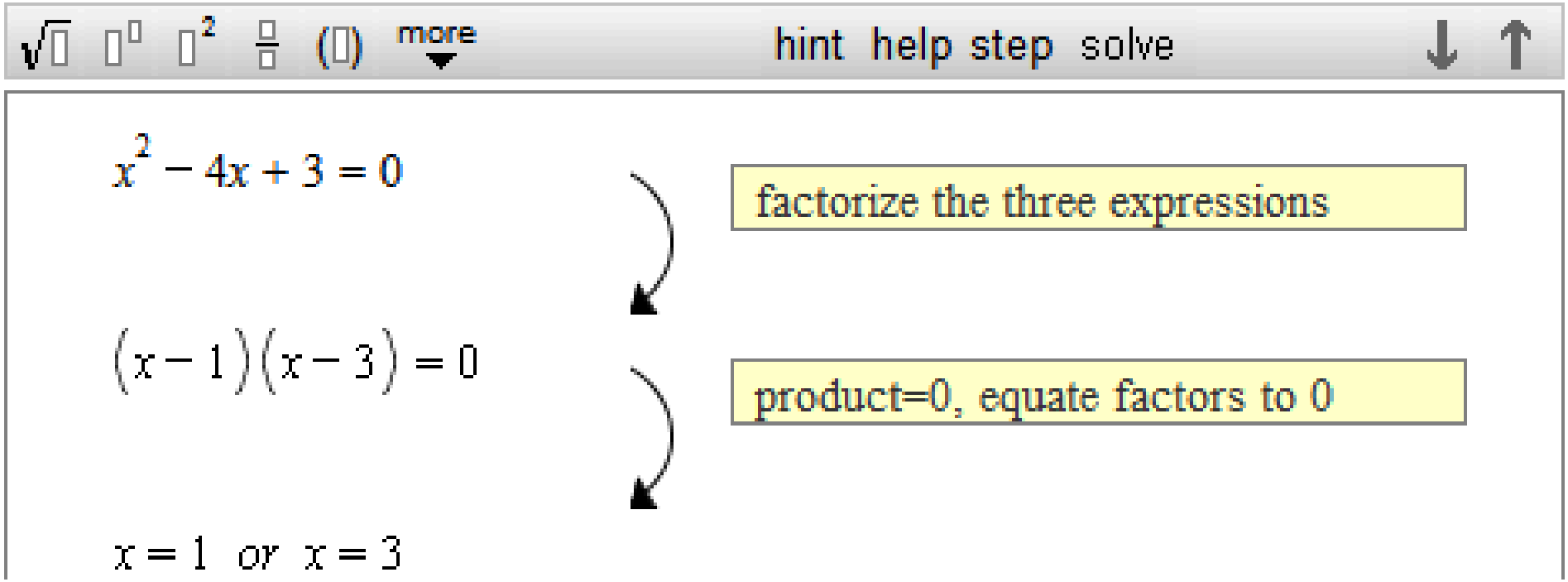
Example of feedback: embedded



Example of feedback: custom feedback



Example of feedback: rule-based feedback



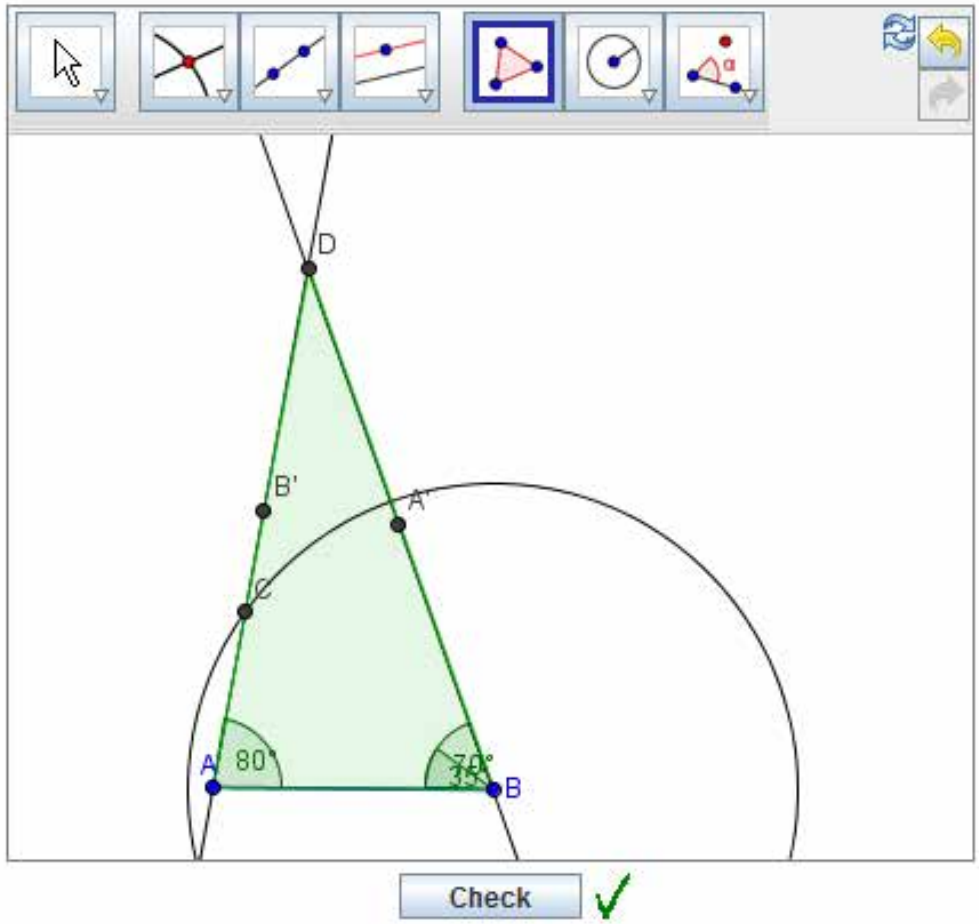
Also geometry checking

Find the triangle

The figure shows a segment *AB*. Construct a triangle *ABC* with the following properties:

- $\angle A = 80°$
- The segment of the bisector from *B* within the triangle has length 4.
- *C* is above *AB*.

Don't forget to draw the requested triangle with the Polygon option!



Page: 1 2 3 4 5 6 7 8 9 10 11

Score: 10 **Total: 10**

More than sum of the parts

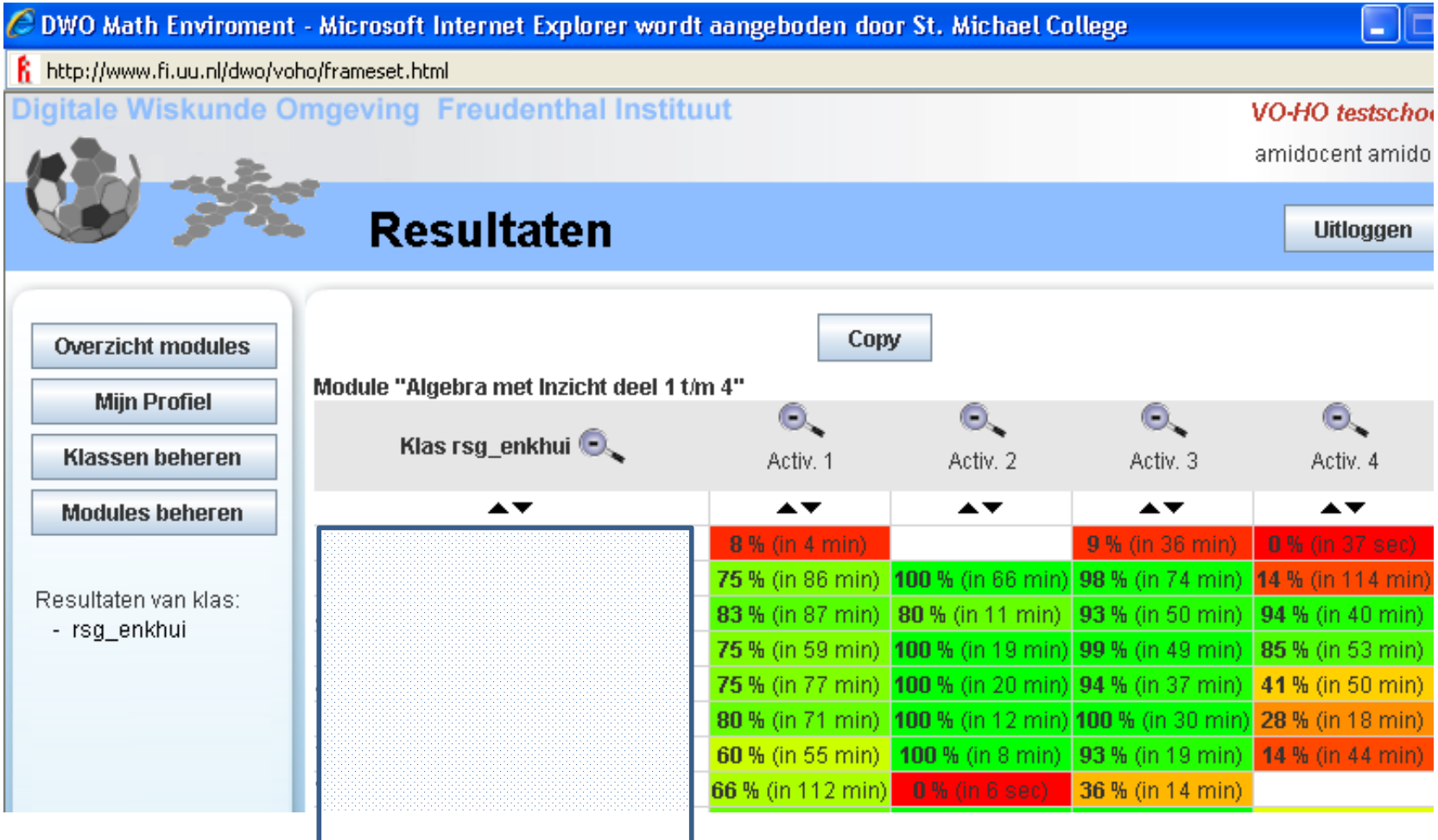
Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. **Storing results**
4. Instructional design
5. Authorability
6. Interoperability and standards

Storing results

- Imagine 10 classes with 30 students and numerous tasks: management system
- But diversity of data types, ranging from simple checkbox to complexity of geometry widget
- Useful for Learning Analytics and studying misconceptions

Store student results, and use these as a teacher to study misconceptions and for starting classroom discussions



More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. Storing results
4. **Instructional design**
5. Authorability
6. Interoperability and standards

(i) students learn a lot from what goes wrong,

(ii) but students will not always overcome these if no feedback is provided, and

(iii) that too much of a dependency on feedback needs to be avoided, as summative assessment typically does not provide feedback.

These three challenges are addressed by principles for **crises**, **feedback** and **fading**, respectively.

Crisis-tasks

"students learn a lot from what goes wrong"

$$(x^2 + 3x - 3)\cdot(8x - 6) = (x^2 + 3x - 3)\cdot(4x + 12)$$

$$8x^3 + 18x^2 - 42x + 18 = 4x^3 + 24x^2 + 24x - 36$$

$$4x^3 - 6x^2 - 66x = -54$$

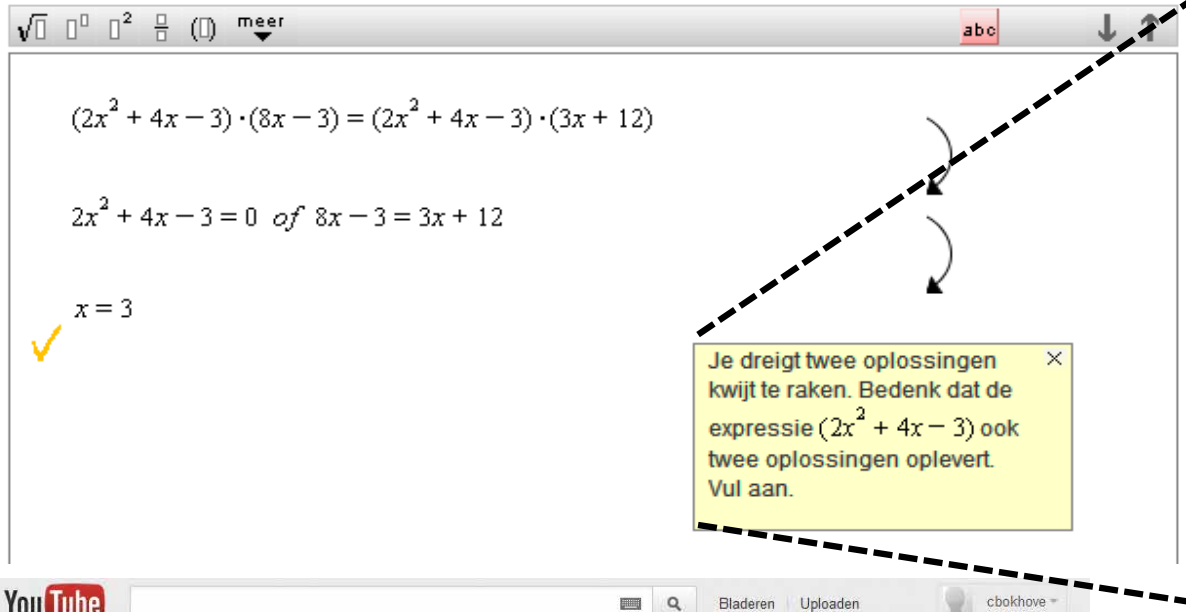
$$4x\left(x^2 - 1\tfrac{1}{2}x - 16\tfrac{1}{2}\right) = -54$$

Je ben goed aan het herschrijven.

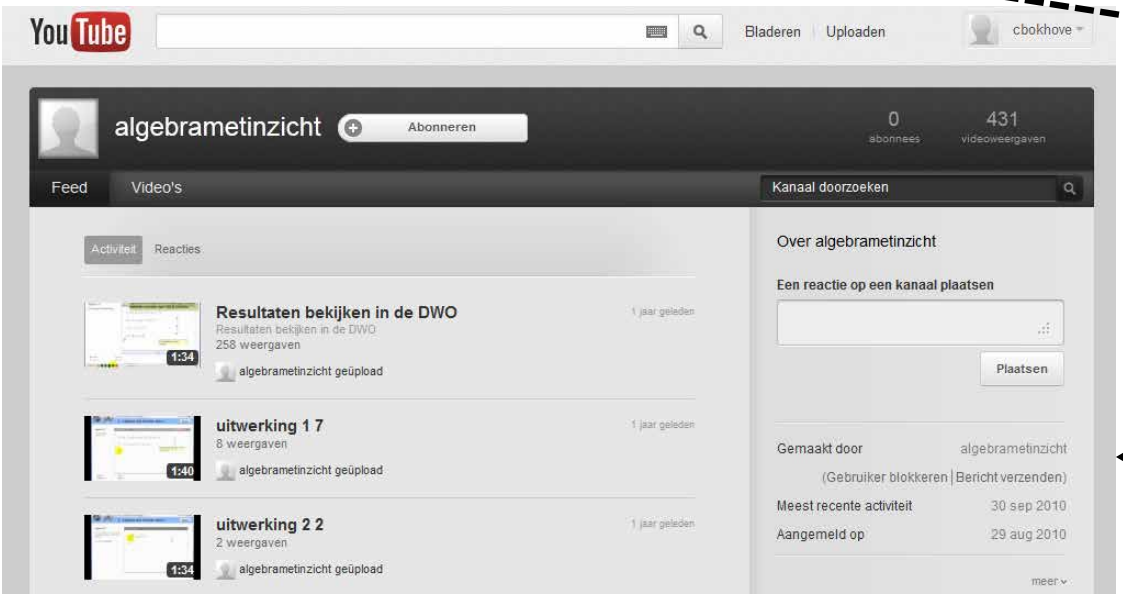
"Failure is, in a sense, the highway to success" - Keats

Feedback: overcoming a crisis

"but students will not always overcome these if no feedback is provided"

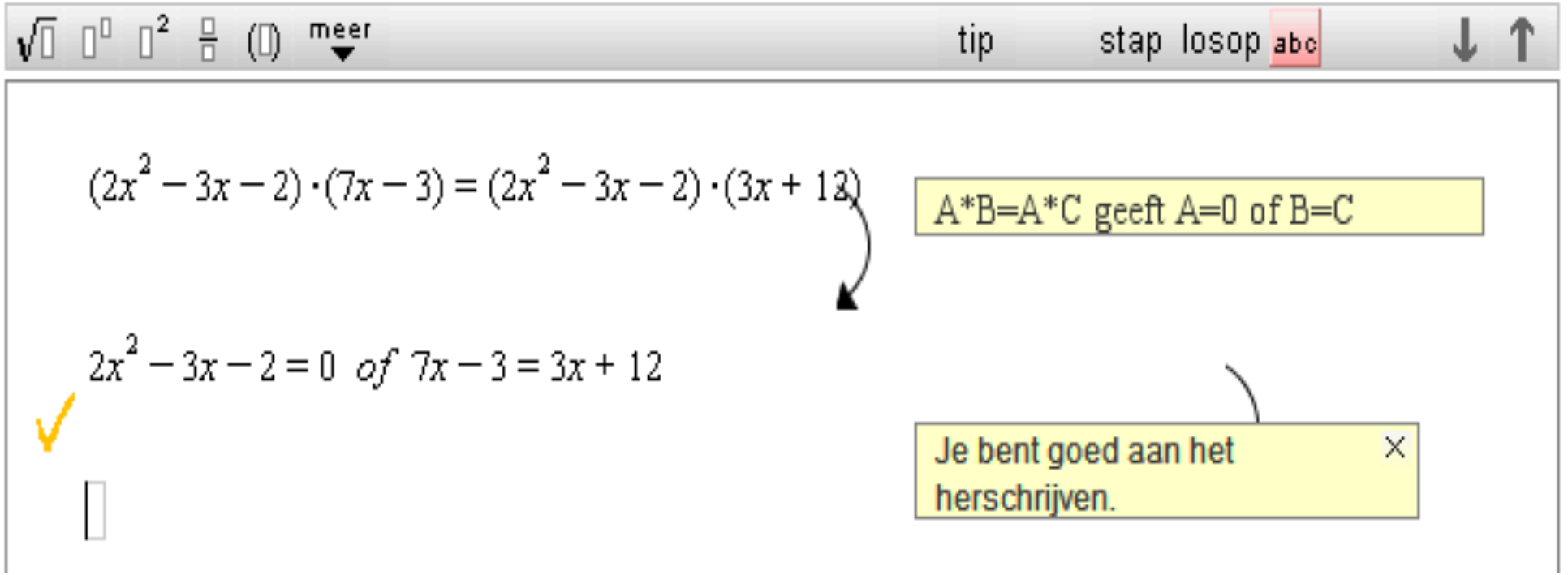


You are about to lose two solutions. Keep in mind that the expression $(2x^2 + 4x - 3)$ also yields two solutions. Please revise.



← Screencast clips

Feedback: worked examples and hints

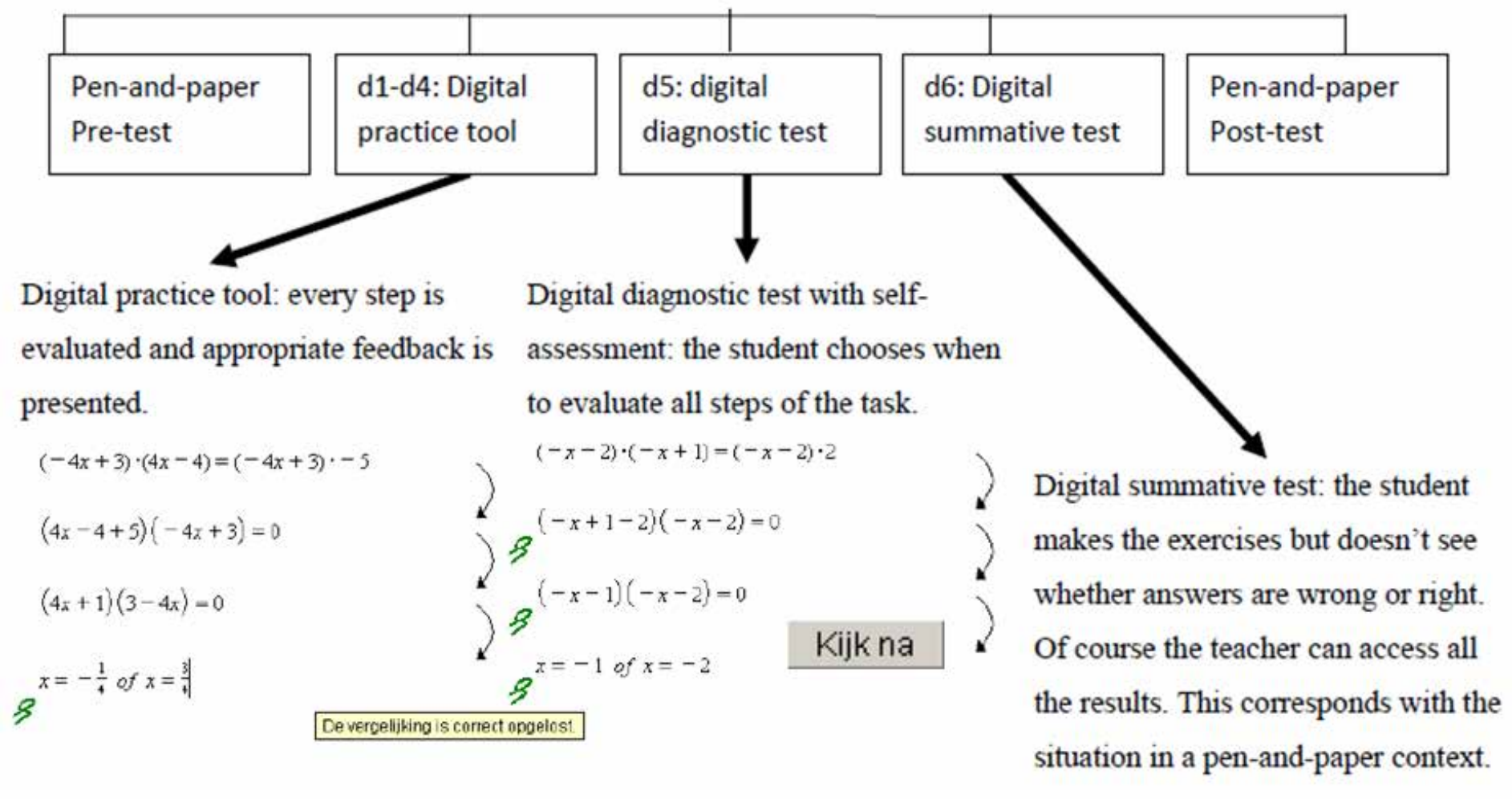


A*B=A*C geeft A=0 of B=C

IDEAS feedback, webservice with Jeuring et al

Fading

"too much of a dependency on feedback needs to be avoided"

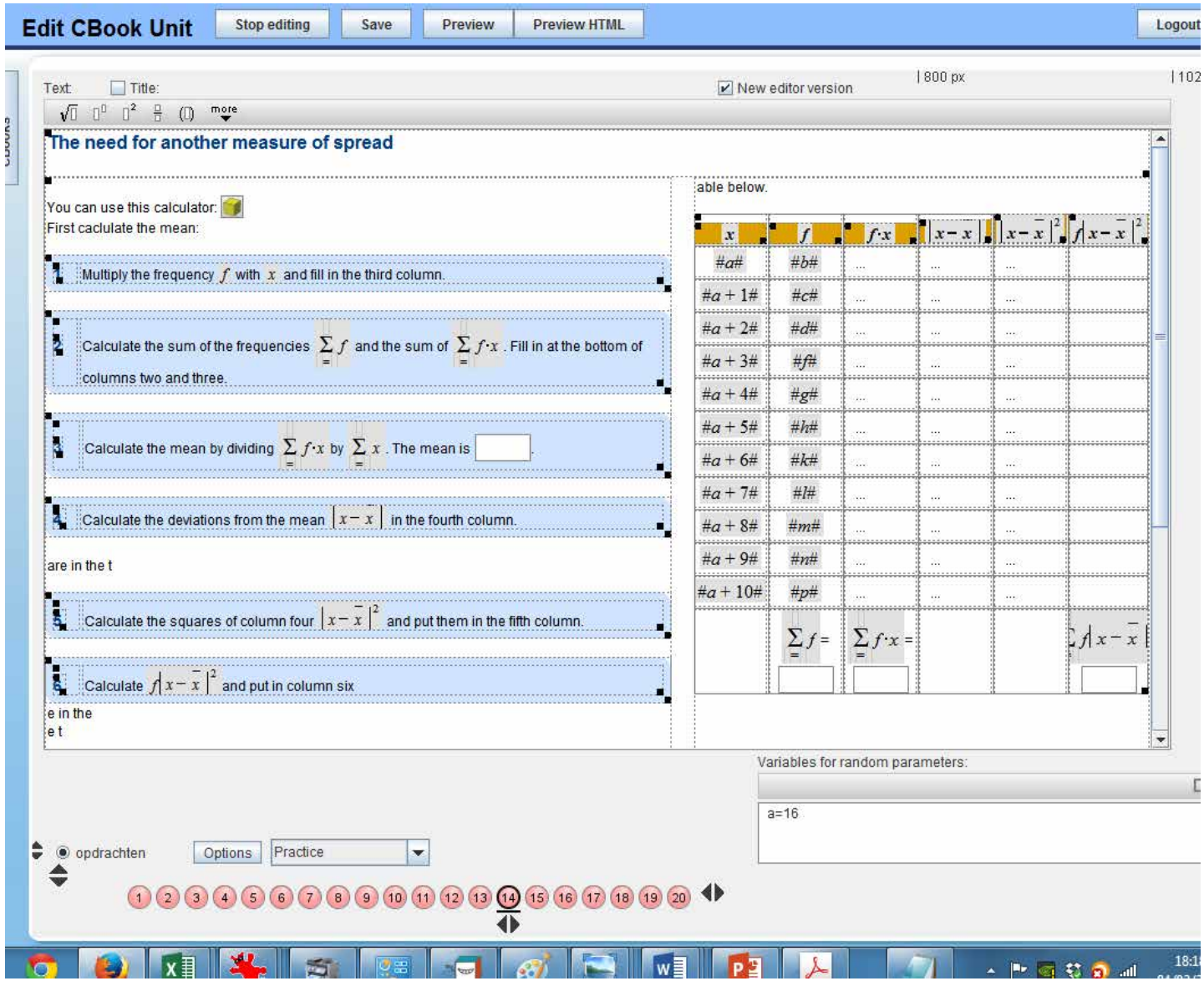


More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. Storing results
4. Instructional design
5. **Authorability**
6. Interoperability and standards

Authorable


Edit CBook Unit
Stop editing
Save
Preview
Preview HTML
Logout
Text:
Title:
New editor version
800 px
The need for another measure of spread
You can use this calculator:
First caclulate the mean:
Multiply the frequency f with x and fill in the third column.
Calculate the sum of the frequencies $\sum f$ and the sum of $\sum f \cdot x$. Fill in at the bottom of columns two and three.
Calculate the mean by dividing $\sum f \cdot x$ by $\sum x$. The mean is
Calculate the deviations from the mean $|x - \bar{x}|$ in the fourth column.
Calculate the squares of column four $|x - \bar{x}|^2$ and put them in the fifth column.
Calculate $f|x - \bar{x}|^2$ and put in column six
able below.
x | f | $f \cdot x$ | $|x - \bar{x}|$ | $|x - \bar{x}|^2$ | $f|x - \bar{x}|^2$
#a# | #b# | ... | ... | ... |
#a + 1# | #c# | ... | ... | ... |
#a + 2# | #d# | ... | ... | ... |
#a + 3# | #f# | ... | ... | ... |
#a + 4# | #g# | ... | ... | ... |
#a + 5# | #h# | ... | ... | ... |
#a + 6# | #k# | ... | ... | ... |
#a + 7# | #l# | ... | ... | ... |
#a + 8# | #m# | ... | ... | ... |
#a + 9# | #n# | ... | ... | ... |
#a + 10# | #p# | ... | ... | ... |
$\sum f =$ | $\sum f \cdot x =$ | $\sum f|x - \bar{x}|$
Variables for random parameters:
a=16
opdrachten
Options
Practice
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

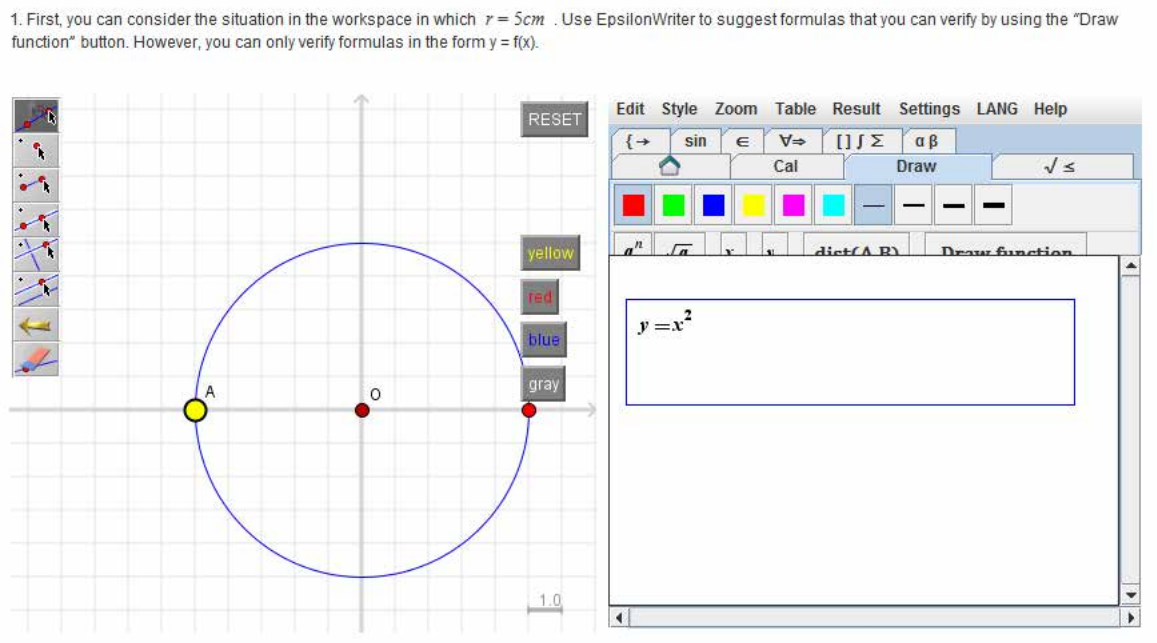
More than sum of the parts

Let me say more on how these digital books could help mathematics education

1. Widgets
2. Interaction (feedback)
3. Storing results
4. Instructional design
5. Authorability
6. **Interoperability and standards**

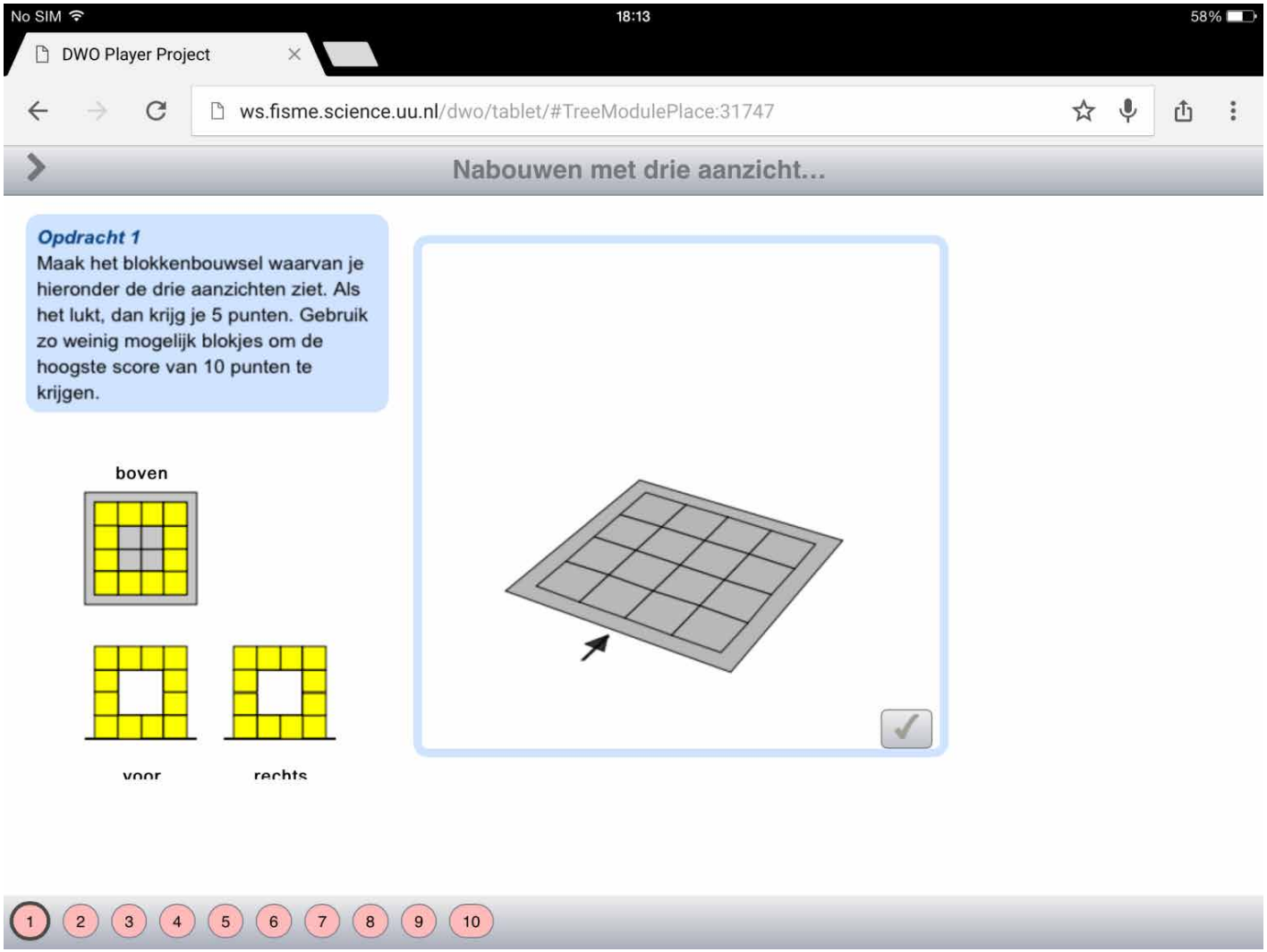
Remains a challenge

- Between the ‘management system’ and individual widgets (e.g. SCORM)
- Between widgets (e.g. multiple representations)



Here, Cinderella (left) can ‘talk’ with Epsilonwriter (right).

HTML5 player

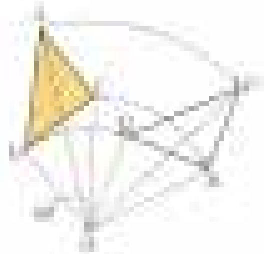


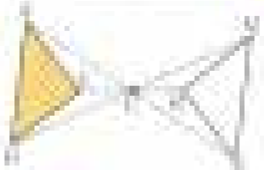
We combined them in the enGasia project

1. Compare geometry education in England, Japan and Hong Kong → some shown now.
2. two digital resources (electronic books) will be designed. They are then implemented in classrooms in those countries.
3. The methodology will include a more qualitative approach based on lesson observations and a quasi-experimental element.

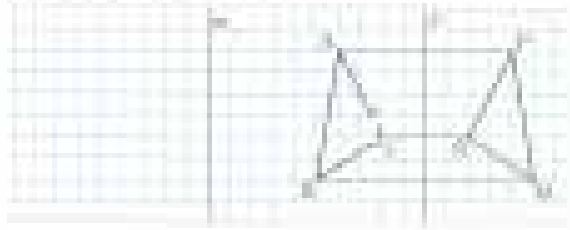
EnGAsia
Technology Enhanced Geometry Education in England, Japan and Hong Kong

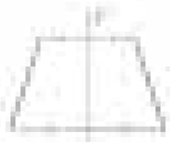
Rotation





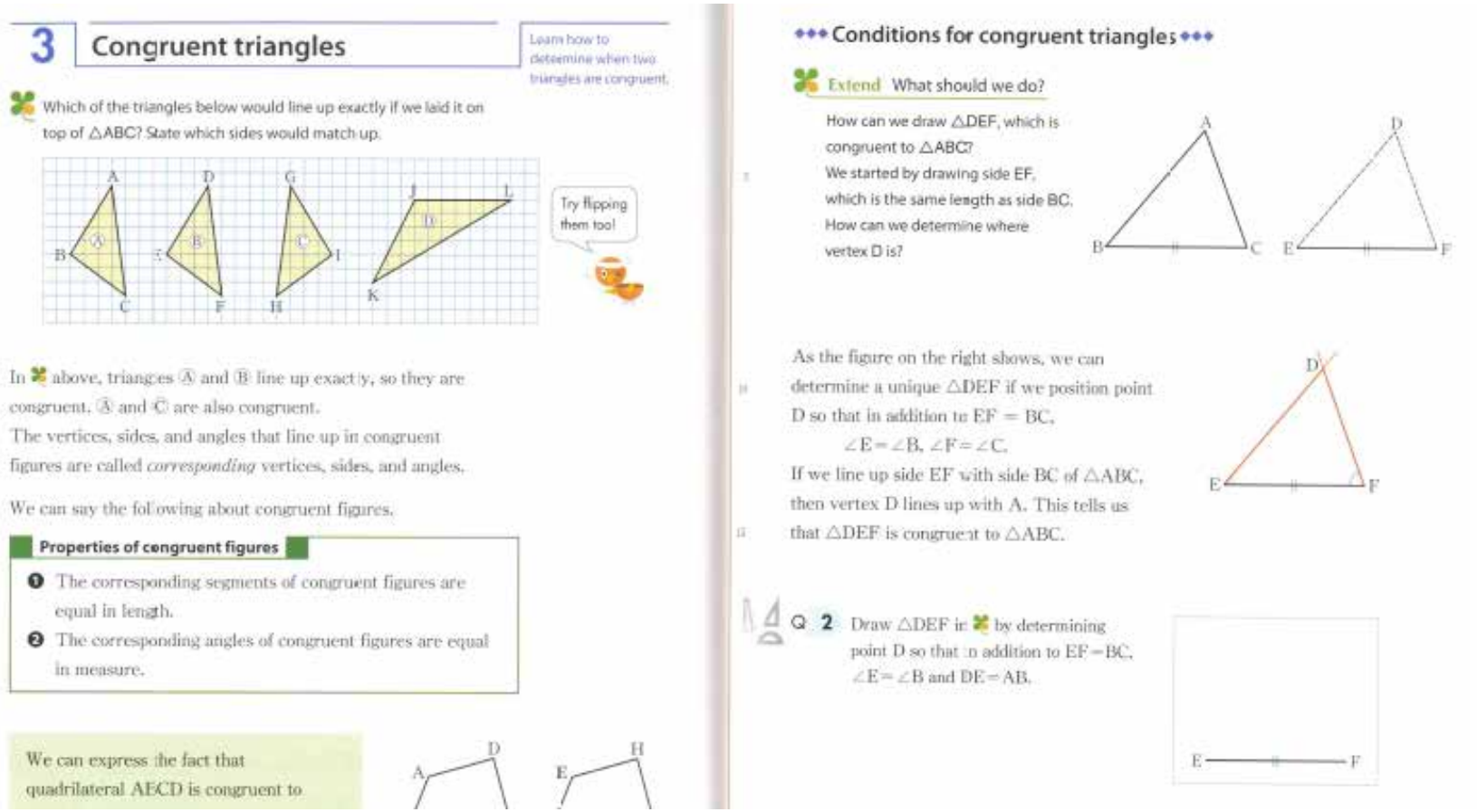
Reflection





The design process

- Multiple widgets
- Some work ‘in the backend’ (e.g. feedback)



3 Congruent triangles

Learn how to determine when two triangles are congruent.

Which of the triangles below would line up exactly if we laid it on top of △ABC? State which sides would match up.

Try flipping them too!

In ✿ above, triangles Ⓐ and Ⓑ line up exactly, so they are congruent. Ⓐ and Ⓒ are also congruent.
The vertices, sides, and angles that line up in congruent figures are called *corresponding* vertices, sides, and angles.

We can say the following about congruent figures.

Properties of congruent figures

❶ The corresponding segments of congruent figures are equal in length.
❷ The corresponding angles of congruent figures are equal in measure.

We can express the fact that quadrilateral AECD is congruent to

◆◆◆ Conditions for congruent triangles ◆◆◆

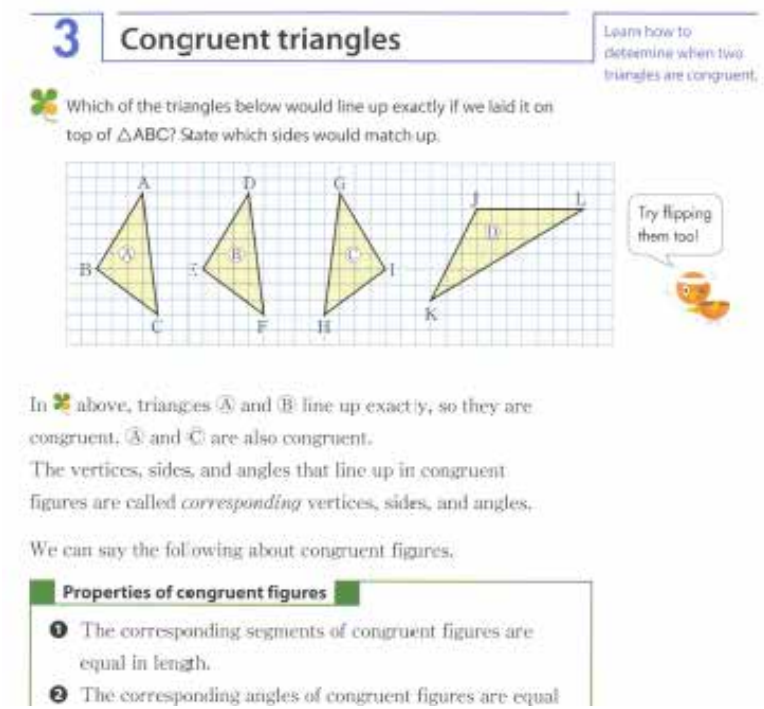
Extend What should we do?

How can we draw △DEF, which is congruent to △ABC?
We started by drawing side EF, which is the same length as side BC.
How can we determine where vertex D is?

As the figure on the right shows, we can determine a unique △DEF if we position point D so that in addition to EF = BC,
∠E = ∠B, ∠F = ∠C.
If we line up side EF with side BC of △ABC, then vertex D lines up with A. This tells us that △DEF is congruent to △ABC.

14 Q 2 Draw △DEF in ✿ by determining point D so that in addition to EF = BC, ∠E = ∠B and DE = AB.

This could be a geogebra widget but perhaps not necessary. More important is feedback.



3 Congruent triangles

Learn how to determine when two triangles are congruent.

Which of the triangles below would line up exactly if we laid it on top of △ABC? State which sides would match up.

In above, triangles Ⓐ and Ⓑ line up exactly, so they are congruent. Ⓐ and Ⓒ are also congruent.

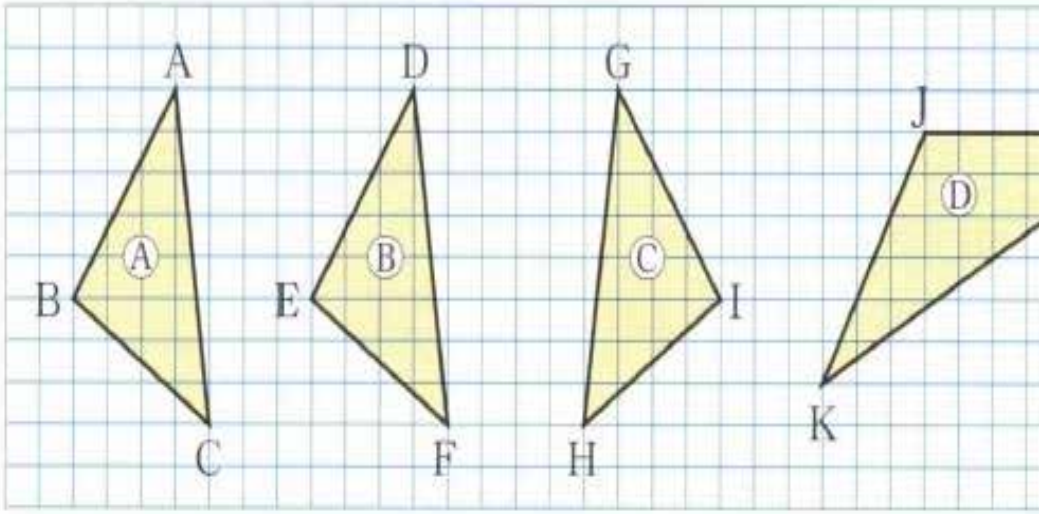
The vertices, sides, and angles that line up in congruent figures are called *corresponding* vertices, sides, and angles.

We can say the following about congruent figures.

Properties of congruent figures

1. The corresponding segments of congruent figures are equal in length.
2. The corresponding angles of congruent figures are equal

Which of the triangles below would line up exactly if we laid it on top of ΔABC ? State which



In the diagram above triangles A and B line up exactly, so they are congruent. A and C are al that line up in congruent figures are called **corresponding** vertices, sides and angles. What

Side AB aligns with sides DE ? and GH ✗

Side BC aligns with sides [ ] and [ ]

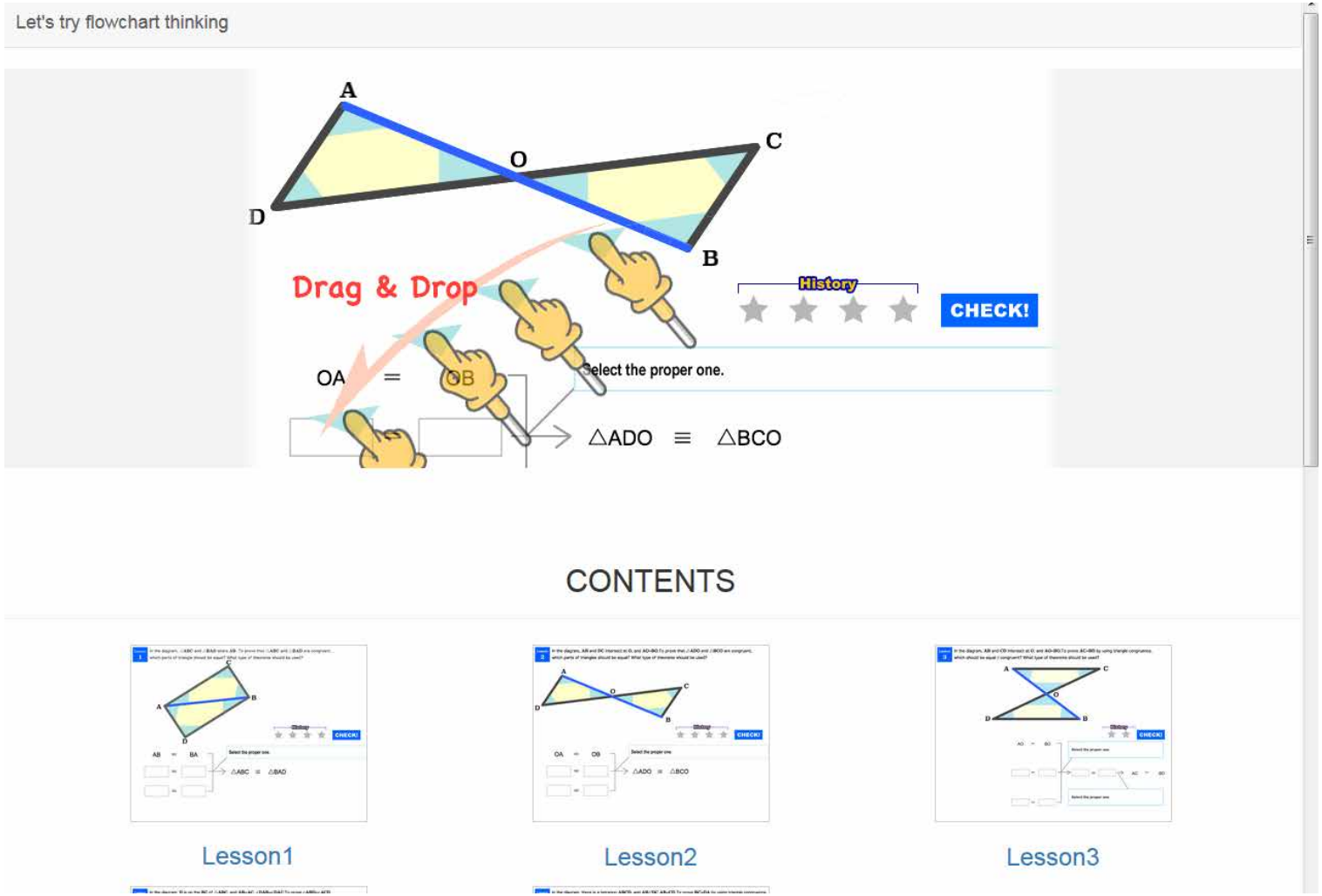
Side CA aligns with sides [ ] and [ ]

You probably thought that the side on the left would be the correct side. However, have a look at the position of the angles. ✗

Page: 1 2 3 4 5 6 7 8

Score: 2 **Total: 2**

Flowchart (Prof. Miyazaki)



http://www.sun-first.jp/fc_html5d/

Conclusions

- Technology sometimes works for mathematics education, and sometimes it doesn't
- Need to think about what you want to achieve
- I showed some examples: feedback, representations, storing student work, etc.
- The enGasia project tries to combine some of these in digital mathematics books
- Only a small part of the possibilities…..